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Seepage to ditches and topographic depressions in saturated and unsaturated soils

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ABSTRACT

An isobar generated by a line or point sink draining a confined semi-infinite aquifer is an analytic curve, to which a steady 2-D plane or axisymmetric Darcian flow converges. This sink may represent an excavation, ditch, or wadi on Earth, or a channel on Mars. The strength of the sink controls the form of the ditch depression: for 2-D flow, the shape of the isobar varies from a zero-depth channel to a semicircle; for axisymmetric flow, depressions as flat as a disk or as deep as a hemisphere are reconstructed. In the model of axisymmetric flow, a fictitious J.R. Philip's point sink is mirrored by an infinite array of sinks and sources placed along a vertical line perpendicular to a horizontal water table. A topographic depression is kept at constant capillary pressure (water content, Kirchhoff potential). None of these singularities belongs to the real flow domain, evaporating unsaturated Gardnerian soil. Saturated flow towards a triangular, empty or partially-filled ditch is tackled by conformal mappings and the solution of Riemann's problem in a reference plane. The obtained seepage flow rate is used as a right-hand side in an ODE of a Cauchy problem, the solution of which gives the draw-up curves, i.e., the rise of the water level in an initially empty trench. HYDRUS-2D computations for flows in saturated and unsaturated soils match well the analytical solutions. The modeling results are applied to assessments of real hydrological fluxes on Earth and paleo-reconstructions of Martian hydrology-geomorphology.

1. Introduction

"I had to live in the desert before I could understand the full value of grass in a green ditch."

Ella Maillart

Analytical models of 2-D seepage towards drainage ditches and trenches, constructed by civil, geotechnical, and agricultural engineers, used the machinery of complex variables (Anderson, 2013, Aravin and Numerov, 1953, Bear, 1972, Kirkham and Powers, 1972, Polubarinova-Kochina, 1962, 1977, hereafter abbreviated as PK-62,77, Skaggs et al., 1999, Strack, 1989, Vedernikov, 1939), in particular, by tackling free boundaries of Darcian flows, the so-called phreatic surfaces.

We recall (see, e.g., Radcliffe and Šimůnek, 2010) that transient, 3-D, saturated-unsaturated flows in porous media (when both water and soil are incompressible) obey the Richards equation:

$$\frac{\partial\theta}{\partial t} = \nabla(K(p)\nabla h) \quad (0)$$

where $\theta(t, x, y, Z)$ is the volumetric moisture content, ∇ is the nabla operator (in Cartesian or cylindrical coordinates), $K(p, x, y, Z)$ is the hydraulic conductivity function, $h(t, x, y, Z) = p + Z$ is the total head, p is the pressure head, Z is a vertical coordinate, and $p(\theta)$ is a capillary pressure (water retention) function, fixed for each soil. Eq. (0) involves the Darsy law $\vec{V} = -K(p)\nabla h$, where $\vec{V}$ is the Darcian flux vector, and the principle of mass conservation.

Boundary value problems (hereafter abbreviated as BVPs) are solved for Eq. (0) by specifying initial conditions, e.g., $\theta(0, x, y, Z)$, as well as imposing physically meaningful boundary conditions (e.g., Dirichlet's, Neumann's). Only numerical codes like HYDRUS-3D (Šimůnek et al., 2016) tackle such problems for arbitrary 3D transient flows. Eq. (0) is a highly nonlinear parabolic PDE. For steady flows, its LHS vanishes, and the equation becomes elliptic. If the flow is purely saturated and the porous medium is homogeneous, then $K(p)=K_s$, where K_s is the sat-

Abbreviations: ADE =, advective dispersion equation; BVP =, boundary value problem; DF =, Dupuit-Forchheimer; LHS, RHS=, left hand side, right hand side; ODE =, Ordinary Differential Equation; OSDP =, Optimal Shape Design Problem; PK-62,77 =, Polubarinova-Kochina, P.Ya., 1962. Theory of Ground Water Movement. Princeton University Press, Princeton. The second edition of the book (in Russian) was published in 1977, Nauka, Moscow.; VG =, van Genuchten.

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