Journal of Computer and System Sciences 112 (2020) 1–12

Contents lists available at ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

Graphs are not universal for online computability ☆

Rodney Downey [a], Matthew Harrison-Trainor [a], Iskander Kalimullin [b], Alexander Melnikov [c,*], Daniel Turetsky [a]

[a] School of Mathematics and Statistics, Victoria University of Wellington, PO Box 600, Wellington, New Zealand
[b] Kazan Federal University, Russian Federation
[c] Massey University Auckland, Private Bag 102904, North Shore, Auckland 0745, New Zealand

ARTICLE INFO

Article history:
Received 6 December 2018
Received in revised form 11 February 2020
Accepted 19 February 2020
Available online 13 March 2020

Keywords:
Computability
Logic
Abstract theory of online computation

ABSTRACT

We show that structures with only one binary function symbol are universal for "online" (punctual) computable structures. In contrast, we give a description of punctually categorical graphs which implies that graphs are *not* universal for online computability.

1. Introduction

The study of general computable processes in algebra is a long tradition going back to the early twentieth century; see van der Waarden [45], Hermann [18], and Dehn [10]. Maltsev [35] and Rabin [39] proposed the following standard model for such investigations: A *computable presentation* of a structure is a coding of the structure with universe $\mathbb{N}$, so that the relations and functions are Turing computable on the codes of elements. For a group this is the same as to say that the group has a recursive presentation with solvable word problem [19,33]. The general area of computable structures now has a well-developed theory. This area has several threads, as can be seen by the large volumes [13,14]. Such investigations rely on the most general notion of an algorithm that we know today, namely Turing computable functions. In particular, we do not assume any time or space bounds in such abstract computations. It is a general phenomenon that many such abstract algorithms can be provably turned into more feasible algorithms for reasons yet to be understood; see, e.g., [17,27]. The present article contributes to the research program which, among other goals, aims to explain this phenomenon.

The other important goal is to give a foundation for online structure theory. There are a large number of natural algorithmic processes which receive input over time but need to provide output promptly. After decades of development, computability theory and computable structure theory give a well-developed framework to investigate the limits of offline computation in infinite mathematics. Nonetheless, there is no such general theory for online potentially infinite structures.

☆ The work of the first and the fourth author was partially supported by the Marsden Fund of New Zealand. The work of the third author was supported by RSF grant no. 18-11-00028, also he was supported by Russian government as a federal professor in mathematics. We thank Noam Greenberg for a fruitful discussion related to the final steps in the proof of the second main result. Noam has declined co-authorship.
* Corresponding author.
E-mail addresses: rod.downey@vuw.ac.nz (R. Downey), matthew.harrisontrainor@vuw.ac.nz (M. Harrison-Trainor), ikalimul@gmail.com (I. Kalimullin), alexander.g.melnikov@gmail.com (A. Melnikov), dan.turetsky@vuw.ac.nz (D. Turetsky).

https://doi.org/10.1016/j.jcss.2020.02.004
0022-0000/