UDC 517

VARIABLE LEBESGUE SPACES AND APPROXIMATION

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A Jackson type inequality of trigonometric approximation and its refinement are obtained for the modulus of smoothness of fractional order in variable exponent Lebesgue spaces $L^{p(x)}$ *with* $1 < essinf_{x\in[0,2\pi]}\, p(x)$, $esssup_{x\in[0,2\pi]}\, p(x) < \infty$.

Keywords: approximation, Lebesgue spaces.

References

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HERMITE-PADÉ APPROXIMANTS FOR A PAIR OF CAUCHY TRANSFORMS WITH INTERLACING SUPPORTS

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Hermite-Padé approximants of type II are vectors of rational functions with common denominator that interpolate a given vector of power series at infinity with maximal order. We are interested in the situation when the approximated vector is given by a pair of Cauchy transforms of smooth complex measures supported on the real line. The convergence properties of the approximants are rather well understood when the supports consist of two disjoint intervals (Angelesco systems) or two intervals that coincide under the condition that the ratio of the measures is a restriction of the Cauchy transform of a third measure (Nikishin systems). In this talk we consider the case where the supports form two interlacing symmetric intervals and the ratio of the measures extends to a holomorphic function in a region that depends on the size of interlacing. This problem was posed and studied by Herbert Stahl at 80-ties, however the detailed proof for the asymptotics of Hermite-Padé approximants has never been published. We shall speak about algebraic functions (of genus 1 and 2) and their abelian integrals (with purely imaginary periods) which defines the main term of asymptotics for this problem.

Keywords: Hermite-Padé approximation, simultaneous interpolation with free poles, non-Hermitian orthogonality, multiple orthogonal polynomials, strong asymptotics.

The talk is based on the joint work [1].

References

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DIRICHLET PROBLEM SOLUTION FOR SIMPLY AND DOUBLY CONNECTED DOMAINS WITH SMOOTH BOUNDARIES

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In this paper we propose a new method for solving the 2D Laplace equation with Dirichlet boundary conditions in simply and doubly connected domains. The method is based on reduction of the problem to the Fredholm integral equation of the second kind. The numerical algorithm connected with truncated Fourier series is applied to convert the Fredholm equation to a finite system of linear equations.

Keywords: Cauchy integral, Fourier polynomial, Dirichlet problem, Fredholm integral equation, simply connected domain, doubly connected domain.

Here we present Cauchy integral method of 2D Dirichlet problem solution for simply and doubly connected domain with smooth boundary. The method is based on reduction of the problem to the Fredholm integral equation of the second kind for the boundary values of the conjugate harmonic function and further conversion of the integral equation to the truncated linear system. The solution of the integral equation has the form of truncated Fourier series. Finally, the solution of the Dirichlet problem has the form of the real part of the Cauchy integral.

The case of simply connected domain

We denote the function to be found as $u(x,y) = \Re B(z)$, where $B(z)$ is analytic in a given simply connected domain Ω. So the problem follows: given the function $f_0(t) = u(x,y)|_{\{x(t),y(t)\}\in\partial\Omega}$, $t \in [0,2\pi]$, while the boundary smooth curve $\partial\Omega$ of the domain Ω passing in a counterclockwise direction, it is necessary to find the function $u(x,y)$, $(x,y)\in\Omega$.

By denoting $g_0(t) = \Im B(z(t))|_{z(t)=x(t)+iy(t)\in\partial\Omega}$, $t\in[0,2\pi]$, and separating the imaginary parts in both the sides of the criterion of the function $f_0(t)+ig_0(t)$ to be the boundary values of the function analytic in Ω, as in [2], the following Fredholm integral equation of the second kind is obtained:

$$g_0(t) = -\frac{1}{\pi}\int_0^{2\pi} f_0(\tau)\left(\log[z(\tau)-z(t)]\right)'_\tau d\tau + \frac{1}{\pi}\int_0^{2\pi} g_0(\tau)\left(\arg[z(\tau)-z(t)]\right)'_\tau d\tau. \quad (1)$$