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Flow polynomials as Feynman amplitudes and their α-representation

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Abstract

© 2017, Australian National University. All Rights Reserved. Let G be a connected graph; denote by τ(G) the set of its spanning trees. Let F q be a finite field, (formula presented), where α e ∈ F q . Kontsevich conjectured in 1997 that the number of nonzero values of s(α, G) is a polynomial in q for all graphs. This conjecture was disproved by Brosnan and Belkale. In this paper, using the standard technique of the Fourier transformation of Feynman amplitudes, we express the flow polynomial F G (q) in terms of the "correct" Kontsevich formula. Our formula represents F G (q) as a linear combination of Legendre symbols of s(α, H) with coefficients (formula presented), where H is a contracted graph of G depending on (formula presented), and |V (H)| is odd.

Keywords

Feynman amplitudes, Flow polynomial, Kontsevich's conjecture, Laplacian matrix, Legendre symbol, Tutte 5-flow conjecture

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