Lobachevskii Journal of Mathematics 2016 vol.37 N4, pages 397-403

Integrable products of measurable operators

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Abstract

© 2016, Pleiades Publishing, Ltd.Let τ be a faithful normal semifinite trace on von Neumann algebra M, $0 < p < +\infty$ and $Lp(M, \tau)$ be the space of all integrable (with respect to τ) with degree p operators, assume also that M is the *-algebra of all τ-measurable operators. We give the sufficient conditions for integrability of operator product A,B M. We prove that $AB \in Lp(M, \tau) \Leftrightarrow AB \in Lp(M, \tau) \Leftrightarrow AB^* \in Lp(M, \tau)$; moreover, $||AB||p = |||A|B||p = |||A||B^*|||p$. If A is hyponormal, B is cohyponormal and $AB \in Lp(M, \tau)$ then $BA \in Lp(M, \tau)$ and $||BA||p \leq ||AB||p$; for $p = 1$ we have $\tau(AB) = \tau(BA)$. A nonzero hyponormal (or cohyponormal) operator $A \in M$ cannot be nilpotent. If $A \in M$ is quasinormal then the arrangement $\mu t(An) = \mu t(A)n$ for all $n \in N$ and $t > 0$. If A is a τ-compact operator and $B \in M$; is such that $|A| \log+|A|$, $ep|B| \in L1(M, \tau)$ then $AB,BA \in L1(M, \tau)$.

http://dx.doi.org/10.1134/S1995080216040041

Keywords

Hilbert space, hyponormal operator, integrable operator, nilpotent, normal trace, projection, quasinormal operator, Radon–Nikodym derivative, rearrangement, state, von Neumann algebra, τ-measurable operator