

Differences of idempotents in C^* -algebras

Bikchentaev A.

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

© 2017, Pleiades Publishing, Ltd. Suppose that P and Q are idempotents on a Hilbert space H , while $Q = Q^*$ and I is the identity operator in H . If $U = P - Q$ is an isometry then $U = U^*$ is unitary and $Q = I - P$. We establish a double inequality for the infimum and the supremum of P and Q in H and $P - Q$. Applications of this inequality are obtained to the characterization of a trace and ideal F -pseudonorms on a W^* -algebra. Let φ be a trace on the unital C^* -algebra A and let tripotents P and Q belong to A . If $P - Q$ belongs to the domain of definition of φ then $\varphi(P - Q)$ is a real number. The commutativity of some operators is established.

<http://dx.doi.org/10.1134/S003744661702001X>

Keywords

C^* -algebra, commutativity, Hilbert space, ideal F -norm, idempotent, linear operator, operator inequality, projection, trace, trace class operator, tripotent, unitary operator, W^* -algebra

References

- [1] Koliha J. J. and Rakočević V., "Invertibility of the difference of idempotents," *Linear Multilinear Algebra*, vol. 51, no. 1, 97–110 (2003).
- [2] Koliha J. J., Rakočević V., and Straškraba I., "The difference and sum of projectors," *Linear Algebra Appl.*, vol. 388, 279–288 (2004).
- [3] Koliha J. J. and Rakočević V., "Fredholm properties of the difference of orthogonal projections in a Hilbert space," *Integral Equations Oper. Theory*, vol. 52, no. 1, 125–134 (2005).
- [4] Bikchentaev A. M., "On idempotent τ -measurable operators affiliated to a von Neumann algebra," *Math. Notes*, vol. 100, no. 4, 515–525 (2016).
- [5] Kalton N. J., "A note on pairs of projections," *Bol. Soc. Mat. Mex. (3)*, vol. 3, no. 2, 309–311 (1997).
- [6] Avron J., Seiler R., and Simon B., "The index of a pair of projections," *J. Funct. Anal.*, vol. 120, no. 1, 220–237 (1994).
- [7] Bikchentaev A. M. and Yakushev R. S., "Representation of tripotents and representations via tripotents," *Linear Algebra Appl.*, vol. 435, no. 9, 2156–2165 (2011).
- [8] Bikchentaev A. M., "Tripotents in algebras: invertibility and hyponormality," *Lobachevskii J. Math.*, vol. 35, no. 3, 281–285 (2014).
- [9] Murphy G., *C^* -Algebras and Operator Theory*, Academic Press, Boston (1990).
- [10] Kadison R. V. and Ringrose J. R., *Fundamentals of the Theory of Operator Algebras. Vol. 1. Elementary Theory*, Acad. Press, New York and London 1983 (Pure Appl. Math.; vol. 100).
- [11] Takesaki M., *Theory of Operator Algebras. I*, Springer-Verlag, New York, Heidelberg, and Berlin 1979.
- [12] Bikchentaev A. M., "Block projection operators in normed solid spaces of measurable operators," *Russian Math. (Iz. VUZ)*, vol. 56, no. 2, 75–79 (2012).
- [13] Topping D. M., "Vector lattices of self-adjoint operators," *Trans. Amer. Math. Soc.*, vol. 115, no. 1, 14–30 (1965).

- [14] Bikchentaev A. M., "Commutativity of projections and characterization of traces on von Neumann algebras," *Sib. Math. J.*, vol. 51, no. 6, 971-977 (2010).
- [15] Bikchentaev A. M., "Commutation of projections and characterization of traces on von Neumann algebras. III," *Int. J. Theor. Phys.*, vol. 54, no. 12, 4482-4493 (2015).
- [16] Bikchentaev A. M., "Inequality for a trace on a unital C^* -algebra," *Math. Notes*, vol. 99, no. 4, 487-491 (2016).
- [17] Koliha J. J., "Range projections of idempotents in C^* -algebras," *Demonstr. Math.*, vol. 24, no. 1, 91-103 (2001).
- [18] Bikchentaev A. M., "Concerning the theory of τ -measurable operators affiliated to a semifinite von Neumann algebra," *Math. Notes*, vol. 98, no. 3, 382-391 (2015).
- [19] Bikchentaev A. M., "On operator monotone and operator convex functions," *Russian Math. (Iz. VUZ)*, vol. 60, no. 5, 61-65 (2016).
- [20] Bikchentaev A. M., "Ideal F -norms on C^* -algebras," *Russian Math. (Iz. VUZ)*, vol. 59, no. 5, 58-63 (2015).