Positivity 2017 vol.21 N1, pages 359-375

L1 -space for a positive operator affiliated with von Neumann algebra

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Abstract

© 2016, Springer International Publishing.In this paper we suggest an approach for constructing an L1-type space for a positive selfadjoint operator affiliated with von Neumann algebra. For such operator we introduce a seminorm, and prove that it is a norm if and only if the operator is injective. For this norm we construct an L1-type space as the complition of the space of hermitian ultraweakly continuous linear functionals on von Neumann algebra, and represent L1-type space as a space of continuous linear functionals on the space of special sesquilinear forms. Also, we prove that L1-type space is isometrically isomorphic to the predual of von Neumann algebra in a natural way. We give a small list of alternate definitions of the seminorm, and a special definition for the case of semifinite von Neumann algebra, in particular. We study order properties of L1-type space, and demonstrate the connection between semifinite normal weights and positive elements of this space. At last, we construct a similar L-space for the positive element of C*-algebra, and study the connection between this L-space and the L1-type space in case when this C*-algebra is a von Neumann algebra.

http://dx.doi.org/10.1007/s11117-016-0422-4

Keywords

C*-algebra, L -space 1, Noncommutative integration, Operator algebra, Positive operator, Semifinite normal weight, Unbounded operator, Von Neumann algebra

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