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L1-space for a positive operator affiliated with von Neumann algebra

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Abstract

© 2016 Springer International PublishingIn this paper we suggest an approach for constructing an (Formula presented.)-type space for a positive selfadjoint operator affiliated with von Neumann algebra. For such operator we introduce a seminorm, and prove that it is a norm if and only if the operator is injective. For this norm we construct an (Formula presented.)-type space as the complition of the space of hermitian ultraweakly continuous linear functionals on von Neumann algebra, and represent (Formula presented.)-type space as a space of continuous linear functionals on the space of special sesquilinear forms. Also, we prove that (Formula presented.)-type space is isometrically isomorphic to the predual of von Neumann algebra in a natural way. We give a small list of alternate definitions of the seminorm, and a special definition for the case of semifinite von Neumann algebra, in particular. We study order properties of (Formula presented.)-type space, and demonstrate the connection between semifinite normal weights and positive elements of this space. At last, we construct a similar L-space for the positive element of C*-algebra, and study the connection between this L-space and the (Formula presented.)-type space in case when this C*-algebra is a von Neumann algebra.

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Keywords

$$L_1$$L1-space, C*-algebra, Noncommutative integration, Operator algebra, Positive operator, Semifinite normal weight, Unbounded operator, Von Neumann algebra