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Makarov's principle for the Bloch unit ball

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Abstract

Makarov's principle relates three characteristics of Bloch functions that resemble the variance of a Gaussian: asymptotic variance, the constant in Makarov's law of iterated logarithm and the second derivative of the integral means spectrum at the origin. While these quantities need not be equal in general, we show that the universal bounds agree if we take the supremum over the Bloch unit ball. For the supremum (of either of these quantities), we give the estimate Σ B 2 < min(0.9,Σ 2), where Σ 2 is the analogous quantity associated to the unit ball in the L ∞ norm on the Bloch space. This improves on the upper bound in Pommerenke's estimate 0.685 2 < Σ B 2 ≤ 1.

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Keywords

Bergman projection, Bloch space, Integral means spectrum, Law of the iterated logarithm

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