

# The enumeration spectrum hierarchy of $\alpha$ -families and $\text{low}_\alpha$ degrees

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## Abstract

© J.UCS. In this paper we introduce a hierarchy of families which can be derived from the integers using countable collections. This hierarchy coincides with the von Neumann hierarchy of hereditary countable sets in the ZFC-theory with urelements from  $\mathbb{N}$ . The families from the hierarchy can be coded into countable algebraic structures preserving their algorithmic properties. We prove that there is no maximal level of the hierarchy and that the collection of non- $\text{low}_\alpha$  degrees for every computable ordinal  $\alpha$  is the enumeration spectrum of a family from the hierarchy. In particular, we show that the collection of non- $\text{low}_\alpha$  degrees for every computable limit ordinal  $\alpha$  is a degree spectrum of some algebraic structure.

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## Keywords

Class of families, Countable family, Degree spectra of structure, Enumeration of family, Low degree  $\alpha$