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Path probability of stochastic motion: A functional approach

Masayuki Hattori [a], Sumiyoshi Abe [a,b,*]

[a] Department of Physical Engineering, Mie University, Mie 514-8507, Japan
[b] Institute of Physics, Kazan Federal University, Kazan 420008, Russia

HIGHLIGHTS

- The concept of path probability of a random walker is studied.
- The general formula for the path probability distribution functional is derived.
- The overdamped limit of the formula is evaluated.
- The probability of finding paths inside a given tube is calculated.
- The theory developed here is applied to the problem of stock price in finance.

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ABSTRACT

The path probability of a particle undergoing stochastic motion is studied by the use of functional technique, and the general formula is derived for the path probability distribution functional. The probability of finding paths inside a tube/band, the center of which is stipulated by a given path, is analytically evaluated in a way analogous to continuous measurements in quantum mechanics. Then, the formalism developed here is applied to the stochastic dynamics of stock price in finance.

1. Introduction

Consider a certain stochastic equation for $X(t)$ that contains an external noise $\eta(t)$. A solution of the equation satisfying a certain initial condition, $X_\eta(t)$, defines a trajectory of a particle/walker. Then, it is essential to determine the probability of finding the particle in the interval $[x,\ x+\mathrm{d}x]$ at time t: $f(x,\ t)\ \mathrm{d}x$, where $f(x,\ t)$ is the probability distribution function given by $f(x,\ t) = \left\langle \delta\left(x - X_\eta(t)\right)\right\rangle_\eta$ with $\langle \bullet \rangle_\eta$ being the average over the noise (see the next section). The passage from the stochastic equation to the probability distribution function is commonly established through the Fokker–Planck equation [1,2]. This is a widely discussed issue that enables one to describe important phenomena such as diffusion and transport.

A less discussed issue may be *path probability*, $P[x]$. In this case, one is concerned with the probability distribution functional that the particle follows the path $x(t)$. One of the first works relevant to this problem may be that in Ref. [3], where irreversible relaxation processes to equilibria are studied for macroscopic thermodynamic variables. Recently, the problem of path probabilities has been revisited in Ref. [4]. It seems, however, that still some points regarding the action functional remain to be clarified in that work.

Our purpose here is to develop the theory of path probabilities in stochastic processes based on the functional method. First, we derive the general formula for the path probability distribution functional associated with the Langevin equation.

* Corresponding author at: Department of Physical Engineering, Mie University, Mie 514-8507, Japan.
E-mail address: suabe@sf6.so-net.ne.jp (S. Abe).

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