

## Nondensity of double bubbles in the D.C.E. degrees

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### Abstract

© Springer International Publishing AG 2017. In this paper, we show that the so-called “double bubbles” are not downward dense in the d.c.e. degrees. Here, a pair of d.c.e. degrees  $d_1 > d_2 > 0$  forms a double bubble if all d.c.e. degrees below  $d_1$  are comparable with  $d_2$ .

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### Keywords

d.c.e. sets, Ershov hierarchy, Exact degrees, Lachlan sets

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