



Contents lists available at ScienceDirect

Journal of Algebra

[www.elsevier.com/locate/jalgebra](http://www.elsevier.com/locate/jalgebra)



## The left and right dimensions of a skew field over the subfield of invariants



Serge Skryabin<sup>1</sup>

*Institute of Mathematics and Mechanics, Kazan Federal University,  
Kremlevskaya St. 18, 420008 Kazan, Russia*

### ARTICLE INFO

#### Article history:

Received 8 July 2016  
Available online 30 March 2017  
Communicated by Nicolás  
Andruskiewitsch

#### MSC:

16T05

#### Keywords:

Hopf algebras  
Hopf module algebras  
Skew fields  
Invariants

### ABSTRACT

If  $H$  is a Hopf algebra and  $A$  an  $H$ -module algebra without nontrivial  $H$ -stable left or right ideals, then the subalgebra of  $H$ -invariant elements  $A^H$  is a skew field and  $A$  may be regarded as a vector space over  $A^H$  with respect to either left or right multiplications. It is proved in the paper that the left dimension of  $A$  over  $A^H$  is equal to the right dimension under the assumptions that  $A$  is semiprimary and  $\dim H < \infty$ . In the case when  $A$  is itself a skew field, this answers a question raised by J. Bergen, M. Cohen and D. Fischman.

© 2017 Elsevier Inc. All rights reserved.

### Introduction

Let  $A$  be a left  $H$ -module algebra where  $H$  is a finite dimensional Hopf algebra over a field  $k$ . In the case when  $A$  is a skew field, i.e., a division ring, the subring of  $H$ -invariant elements  $A^H$  is a skew field too, and both the left dimension  $[A : A^H]_l$  and the right dimension  $[A : A^H]_r$  of  $A$  over  $A^H$  are finite. In a 1990 paper Bergen, Cohen

*E-mail address:* [Serge.Skryabin@kpfu.ru](mailto:Serge.Skryabin@kpfu.ru).

<sup>1</sup> This work was funded by the subsidy allocated to Kazan Federal University for the state assignment in the sphere of scientific activities.