

Dominated convergence in measure on semifinite von Neumann algebras and arithmetic averages of measurable operators

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Abstract

Consider a von Neumann algebra M with a faithful normal semifinite trace τ . We prove that each order bounded sequence of τ -compact operators includes a subsequence whose arithmetic averages converge in τ . We also prove a noncommutative analog of Pratt's lemma for $L^1(M, \tau)$. The results are new even for the algebra $M = B(H)$ of bounded linear operators with the canonical trace $\tau = \text{tr}$ on a Hilbert space H . We apply the main result to $L^p(M, \tau)$ with $0 < p \leq 1$ and present some examples that show the necessity of passing to the arithmetic averages as well as the necessity of τ -compactness of the dominant. © 2012 Pleiades Publishing, Ltd.

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Keywords

arithmetic average, Banach space, Banach-Saks property, Hilbert space, measurable operator, normal semifinite trace, spectral theorem, topology of convergence in measure, von Neumann algebra