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On the structure of finite coverings of compact connected groups

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Abstract

Finite-sheeted covering mappings onto compact connected groups are studied. We show that for a covering mapping from a connected Hausdorff topological space onto a compact (in general, non-abelian) group there exists a topological group structure on the covering space such that the mapping becomes a homomorphism of groups. To prove this fact we construct an inverse system of covering mappings onto Lie groups which approximates the given covering mapping. As an application, it is shown that a covering mapping onto a compact connected abelian group G must be a homeomorphism provided that the character group of G admits division by degree of the mapping. We also get a criterion for triviality of coverings in terms of means and prove that each finite covering of G is equivalent to a polynomial covering.

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0. Introduction

This paper deals with finite-sheeted covering mappings onto compact connected groups. Particular attention is paid to covering mappings onto abelian groups.

In studying covering mappings from topological spaces onto topological groups it is natural to regard the problem on the existence of a topological group structure on a covering space relative to which a given covering mapping becomes a homomorphism of groups. It follows from classical properties of lifting mappings to covering spaces that if a covering space is connected and locally path connected, then this problem has a positive solution, i.e., the desired structure exists (see, e.g., [31, Theorem 79]). In that case we say that the structure of topological group lifts to a covering space, and results of that kind are called the covering group theorems. In this paper we prove the covering group theorem for a finite-sheeted covering mapping from a connected Hausdorff topological space onto a compact connected (in general, non-abelian) group (Theorem 1). Note that we do not suppose that the group is locally connected. In order to prove Theorem 1 we first construct a family of k-fold covering mappings onto Lie groups which

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