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Sharp Hardy-type inequalities with Lamb's constants

Avkhadiev F., Wirths K.
Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

Let Ω be an n-dimensional convex domain with finite inradius δ 0 = sup xεΩ δ, where δ = dist(x,ρΩ), and let (p,q) be a pair of positive numbers. For functions vanishing at the boundary of the domain and any v € [0, p/q] we prove the following Hardy-type inequality √Ω|▽f| 2dx/ δ p-1&ge h √Ω|f| 2dx/δ p+1+λ 2/δ q 0√ Ω|f| 2dx/δ p-q+1 with two sharp constants h =p 2-v 2q 2/4&ge 0 and λ=q/ 2λ v(2p/q)>0, where z = λ v(p) is the Lamb constant defined as the first positive root of the equation pj v(z) + 2zJ' v(z) = 0 for the Bessel function J v., We prove that z = λ v(p) as a function in p can be found as the solution of an initial value problem for the differential equation dz/dp = 2z/p 2-4v 2+4z 2. For " = 1 our inequality is an improvement of the original Hardy inequality for finite intervals. For n > 1 and p = q/2 = lit gives a new sharp form of the Hardy-type inequality due to H. Brezis and M. Marcus. The case h = 0, v = 1/2, p = 1 and q = 2 coincides with sharp eigenvalue estimates due to J. Hersch for n = 2, and L. E. Payne and I. Stakgold for n ≥ 3.

Keywords

Bessel function, Convex domain, First eigenvalue, Hardy inequality, Inradius, Lamb constant