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To the theory of operator monotone and operator convex functions

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Abstract

We prove that a real function is operator monotone (operator convex) if the corresponding monotonicity (convexity) inequalities are valid for some normal state on the algebra of all bounded operators in an infinite-dimensional Hilbert space. We describe the class of convex operator functions with respect to a given von Neumann algebra in dependence of types of direct summands in this algebra. We prove that if a function from $\mathbb{R}+$ into $\mathbb{R}+$ is monotone with respect to a von Neumann algebra, then it is also operator monotone in the sense of the natural order on the set of positive self-adjoint operators affiliated with this algebra. © 2010 Allerton Press, Inc.

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Keywords

C*-algebra, operator convex function, operator monotone function, von Neumann algebra