

# Affiliated subspaces and the structure of von neumann algebras

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## Abstract

The interplay between order-theoretic properties of structures of subspaces affiliated with a von Neumann algebra  $M$  and the inner structure of the algebra  $M$  is studied. The following characterization of finiteness is given: a von Neumann algebra  $M$  is finite if and only if in each representation space of  $M$  one has that closed affiliated subspaces are given precisely by strongly closed left ideals in  $M$ . Moreover, it is shown that if the modular operator of a faithful normal state  $\varphi$  is bounded, then all important classes of affiliated subspaces in the GNS representation space of  $\varphi$  coincide. Orthogonally closed affiliated subspaces are characterized in terms of the supports of normal functionals. It is proved that complete affiliated subspaces correspond to left ideals generated by finite sums of orthogonal atomic projections. © Theta, 2013.

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## Keywords

Modular theory, States and weights on von neumann algebras, Subspaces affiliated with a von neuman algebra