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# Streets and pedestrian trajectories in an urban district: Bejan's constructal principle revisited

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## HIGHLIGHTS

- In an urban district a citizen travels along a pedestrian and public transportation segment.
- Starting points are uniformly distributed over a rectangle, a vertex is the endpoint.
- Minimal-energy trajectories are found.
- Optimal district sizes–shapes are explicitly calculated by elementary calculus.

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## ABSTRACT

Bejan's (1996) analytical optimum for the average travel time between an arbitrary point in a rectangle and its "gravitating" vertex along a trajectory, consisting of a low-resistance side of the rectangle (street) and a high-resistance internal segment perpendicular to this side, is generalized in the following way: (a) our street length is shorter than the rectangle side; (b) our "internal" segment of the trajectory is tilted with respect to the street at an angle which is either constant (a middle zone of the district) or varies in two other zones where trajectories directly converge to either the vertex or the tip of the street, wherefrom a pedestrian embarks a bus transporting him to the vertex; (c) our double integrals in the criterion and the first–second derivative tests are more cumbersome for evaluation and require computer algebra for calculation. The total energy spent by a moving entity is assumed to be linearly dependent on the length of the walkway or street trajectory segments with two different resistance coefficients. This energy is minimized for an individual particle such that the tilt angle is determined and zonation is done. Then another minimum of the area-averaged "community" energy is searched with the total rectangle area as a constraint and the side ratio of the rectangle as a control variable. Explicit analytical expressions for double integrals and minima are presented. Alternative criteria, involving the travel time, non-constant specific bus fare and area-averaged tilt angle are discussed.



*"Größe heißt: Richtung-geben."*

F. Nietzsche. *Menschliches, Allzumenschliches*

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