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'Fractional' kinetic equations and 'universal' decoupling of a memory function in mesoscale region

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Abstract

It is proved that kinetic equations containing non-integer integrals and derivatives appear in the result of reduction of a set of micromotions to some averaged collective motion in the mesoscale region. In other words, it means that after a proper statistical average the microscopic dynamics is converted into a collective motion in the mesoscopic regime. A fractal medium containing weakly and strongly correlated relaxation units has been considered. It is shown that in most cases the original of the memory function recovers the Riemann–Liouville integral. For a strongly correlated fractal medium a *generalization* of the Riemann–Liouville integral is obtained. For the fractal-branched processes one can derive the stretched-exponential law of relaxation that is widely used for description of relaxation phenomena in disordered media. It is shown that the generalized stretched-exponential function describes the averaged collective motion in the fractal-branched complex systems. It is confirmed that the generalized stretched-exponential law can be definitely applicable for description of a set of spontaneous/evoked postsynaptic signals in the living matter. The application of these fractional kinetic equations for the description of the dielectric relaxation phenomena is also discussed. These kinetic equations and their generalizations can be applicable for the description of different relaxation or diffusion processes in the intermediate (mesoscale) region.

Keywords: Memory function; Kinetic equations with non-integer operators of differentiation/integration containing real and complex-conjugate power-law exponents; The generalized Riemann–Liouville integral; Linear response and Mori–Zwanzig kinetic equations; The 'universal' decoupling procedure

1. Introduction

Recently, much attention has been paid to the existence of equations containing non-integer operators with *real* fractional exponent [1–7]. But in papers related to consideration of the fractional equations containing non-integer operators of integration or differentiation are realized on an 'intuitive' level in the form of some postulates/suppositions *imposed* on a structure or model considered. At present, a systematic deduction of kinetic equations containing non-integer integration/differentiation operators from a structure of a disordered medium with the usage of the methods of non-equilibrium statistical mechanics is *absent*. So, there is the barest necessity to derive kinetic equations with non-integer operators of differentiation and integration from

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