



Analytical investigation of the specific heat for the Cantor energy spectrum



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ABSTRACT

For the energy spectrum obtained for monoscale Cantor set the correct analytical calculations of the specific heat in the frame of the Boltzmann–Maxwell statistics have been performed. These evaluations were realized with the help of Mellin's transform. The accurate analytical expressions for the specific heat in all temperature range were obtained. They demonstrate the log-periodic behavior in low-temperature and non-oscillatory behavior in high-temperature regions, accordingly. The accurate value of the limiting temperature determining the boundary between these two regions was found and evaluated.

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1. Introduction

The experimental discovery of quasi-crystals by Shechtman et al. [1] produced a great interest in the understanding of the properties of these systems, as was shown later by the great amount of theoretical and experimental work that followed. The fact that they are in some sense midway between disorder (many of their physical properties exhibit an erratic appearance) and order (their definition, and construction, follows purely deterministic rules) makes them attractive objects of research. Since their first experimental realization in quasi-periodic GaAs–AlAs heterostructures in 1985 by Merlin and collaborators [2], their interest has only increased. More specifically, the Molecular Beam Epitaxy technique has produced and driven a multiplication of possible such structures (Fibonacci, Thue–Morse, double-period sequences; other possibilities could be Cantor sets, prime numbers, etc.). The behavior of a variety of particles and quasi-particles (electrons, photons, plasmon-polaritons, magnons) in quasi-periodic sequences has been and is currently being studied [3–11]. Now, there is a common feature which can be considered as the basic signature of such structures, and this is a fractal energy spectrum. These spectra tend, however, to be quite complex. In order to enlighten the thermodynamic consequences of fractal energy spectra, in Refs. [12,13] and [14], one- and multiscale fractal energy spectra were studied within Boltzmann statistics. It was shown that the scale invariance of the spectrum has strong consequences on the thermodynamical quantities. In particular, the specific heat oscillates log-periodically as a function of the temperature. Moreover,

general scaling arguments and a detailed analysis of the integrated density of states allowed for a quantitative prediction of the average value (which is related to the average density of states), period and amplitude of the oscillations. Moreover, these results were extended to N -particle systems described by quantum statistics [15]; it was shown that for phonons, and for bosons in general, the Boltzmann scenario survives the inclusion of quantum symmetries. The fermionic case is more delicate, however, in some special cases, log-periodic oscillations can still be observed.

The aim of this present work is to demonstrate analytical calculations of the specific heat in the frame of the Maxwell–Boltzmann statistics for fractal energy spectrum obtained from monoscale Cantor set in *all* temperature range. The possibility of realization of the correct calculations is based on the general formula for the elements of the Cantor set that has been used in papers [12,14]. In these papers for analytical proof of existence of the log-periodic oscillations that appeared in the specific heat behavior for the simplest case of monoscale energy spectrum of the Cantor set the Poisson summation formula was used. But in the frame of this approach the region of log-periodic behavior was not evaluated properly and expression for the specific heat out of this region was also not shown. But with the help of Mellin's transform it becomes possible to determine the limits of the log-periodic region and find analytical expressions for the specific heat in these two oscillation/non-oscillation regimes. In the given paper with the usage of Mellin's transformation it becomes possible to realize accurate analytical calculations for the specific heat associated with more general monoscale Cantor set in comparison with results considered earlier in paper [14]. It is necessary to note that approach based on Mellin's transform is rather productive and it was applied with success for the solution of the problems related to the anomalous dielectric relaxation [16,17], thermodynamics of spin

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