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# Entropy of an extremal electrically charged thin shell and the extremal black hole

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ABSTRACT

There is a debate as to what is the value of the entropy $S$ of extremal black holes. There are approaches that yield zero entropy $S=0$, while there are others that yield the Bekenstein–Hawking entropy $S=A_+/4$, in Planck units. There are still other approaches that give that $S$ is proportional to $r_+$ or even that $S$ is a generic well-behaved function of $r_+$. Here $r_+$ is the black hole horizon radius and $A_+=4\pi r_+^2$ is its horizon area. Using a spherically symmetric thin matter shell with extremal electric charge, we find the entropy expression for the extremal thin shell spacetime. When the shell's radius approaches its own gravitational radius, and thus turns into an extremal black hole, we encounter that the entropy is $S=S(r_+)$, i.e., the entropy of an extremal black hole is a function of $r_+$ alone. We speculate that the range of values for an extremal black hole is $0 \le S(r_+) \le A_+/4$.



## 1. Introduction

The entropy $S$ and thermodynamics of black holes have been worked out first by Bekenstein [1] and Hawking and collaborators [2,3]. The Bekenstein–Hawking entropy is given by $S=A_+/4$, where $A_+=4\pi r_+^2$, $A_+$ and $r_+$ are the horizon area and the horizon radius, respectively, and we are putting all the natural constants equal to one, i.e., we use Planck units. York and collaborators [4–6] (see also [7,8]) have further worked out the black hole thermodynamic properties by using canonical and grand canonical thermodynamic ensembles. There are several other methods that can be used to study black hole thermodynamics, one that suits us here uses matter shells [9–11]. In this method, one studies the generic thermodynamics of the shells at any shell radius, and as one sends the shell to its own gravitational radius one recovers the $S=A_+/4$ Bekenstein–Hawking entropy. This is the quasiblack hole method, the evident power of it was displayed in [12].

A particular class of black holes is the extremal black hole class. Electrically charged black holes in general relativity, the ones we are interested here, have $m \ge Q$, and the extremal black holes are characterized by having their mass $m$ equal to their electric charge $Q$, $m=Q$. The extremal black holes seem to have distinct properties. For instance, according to the Hawking temperature formula, extremal black holes have zero temperature. In addition the entropy of an extremal black hole is a subject of a wide debate as there are different reasonings that can be applied which lead to different values for the entropy. Hawking and collaborators [13] and Teitelboim [14] have given topological arguments which point to the conclusion that extremal black holes have zero entropy. Further evidence from other arguments for $S=0$ for extremal black holes was provided in [15–17], see also [18,19]. One could also argue, naively, that since the Hawking temperature is zero, then according to one of the formulations of the third law of thermodynamics as many textbooks present it should have zero entropy.

However, there remain doubts why the Bekenstein–Hawking formula does not hold. After all, working out the entropy of non-extremal black holes and taking the extremal limit $m=Q$ yields $S=A_+/4$, see, e.g., [2,3,5,10]. In this case, the thermodynamic argument would not hold, the extremal black hole could be a system of minimum energy and degenerate ground state and such systems can have entropy even at zero temperature. Moreover, in string theory, there are arguments, other than geometrical, that make use of a direct counting of string and D-brane states in composite

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