Nonlinear Analysis 92 (2013) 177–182

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Application of penalty methods to non-stationary variational inequalities☆

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ARTICLE INFO

Article history:
Received 11 April 2013
Accepted 13 July 2013
Communicated by S. Carl

MSC:
90C33
47J20
49J40
65K15
65J20

Keywords:
Variational inequality
Non-stationarity
Non-monotone mappings
Approximation sequence
Penalty method
Regularization
Coercivity conditions

ABSTRACT

We solve a general variational inequality problem in a finite-dimensional setting, where only approximation sequences are known instead of exact values of the cost mapping and feasible set. We suggest to utilize a sequence of solutions of auxiliary problems based on a penalty method. Its convergence is attained without concordance of penalty and approximation parameters under mild coercivity type conditions. We also show that the regularized version of the penalty method enables us to further weaken the coercivity condition.

1. Introduction

Let D be a nonempty set in the n-dimensional Euclidean space $\mathbb{R}^n$ and let $G : \mathbb{R}^n \to \mathbb{R}^n$ be a mapping. Then we can define the *variational inequality* (VI for short): Find $x^* \in D$ such that

$$\langle G(x^*), x - x^* \rangle \geq 0 \quad \forall x \in D. \tag{1}$$

VIs give a suitable common format for a lot of problems arising in Economics, Mathematical Physics, and Operations Research and are closely related with other general problems in Nonlinear Analysis, such as fixed point, optimization, complementarity, and equilibrium problems. Now VI can be treated as a differential form of equilibrium conditions in complex systems; see, e.g., [1–3] and the references therein.

However, exact values of the cost mapping G and feasible set D may be unknown for many real problems. On the one hand, this is clearly invoked by the usual calculation errors and incompleteness of information about the problem under solution. On the other hand, it might be useful to replace the initial problem by a sequence of auxiliary ones with better properties, as in regularization and penalty methods. Therefore, one has only sequences of some approximations for G and

☆ This work was supported in part by the RFBR grants, project Nos. 13-01-00029a and 13-01-00368a.
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0362-546X/$ – see front matter
http://dx.doi.org/10.1016/j.na.2013.07.015