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An $\mathbb{R}$-linear conjugation problem for a plane two-component heterogeneous structure with an array of periodically distributed sinks/sources

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ABSTRACT

An $\mathbb{R}$-linear conjugation problem for a planar structure consisting of an isotropic strip and adjacent half-plane with contrasting permeabilities is solved. The whole structure is bounded from above by an equipotential line. An exact analytical solution is derived in terms of complex velocity in the class of one-periodical piece-wise meromorphic functions. Their principal part is the sum of periodically distributed simple poles, fixed in advance. Cases with singularities internal to the strip and subjacent half-plane are distinguished from a special case of poles positioned along the interface.

1. Introduction

Mathematical sinks and sources (poles) model horizontal, vertical wells and drains placed in rock/soil, with applications in groundwater hydrology, reservoir and agricultural engineering (see e.g. [1–4]). The generated fields of flow velocities depend on the number of wells, the distance between them, pumping/injection rates, hydraulic heads/pressures within borehole and the heterogeneity of the near-well zone (formation damage, skin effect [5]) and of the permeability of geological strata (soil layers) in which the wells are placed. The case of installing a drain or well in the substratum of a two-layered porous system has been studied in [6,1,7]. Here we extend their solutions to a periodic drainage. Moreover, in smartly designed bi-level agricultural drainage schemes [8] two arrays of drains are constructed such that within one period the upper drain commands over the lower drain. In petroleum industry the injection-abstraction wells are drilled also arbitrarily with respect to strata boundaries. So, in this paper we consider the most general case of an arbitrary number of sinks and sources positioned arbitrarily with respect to the interface between an upper stratum and substratum and an equipotential from which the fluid is supplied to the wells (ponded soil surface, quasi-horizontal water table or a quasi-static oil–water contact boundary in secondary recovery). The singularities (wells) can make periodic arrays (two as in [8] or any other number).

Thus, the present paper is a mathematical generalization of a recent work [9] where periodic drains in a two-layered soil were studied, with a limitation that the sinks are placed in the upper stratum just under an equipotential horizon (ponded soil surface). Similarly to the scheme in [9] a half–strip makes one period of the whole network of singularities. The sinks/sources are, however, arbitrary situated within this half-strip. In other words, a finite group of sinks/sources is infinitely but $2L$-periodically repeated as is shown in Fig. 1. Our aim is to find an exact analytical solution of the problem and using it to reconstruct a flow net in the structure. It is worth to say that at the present moment there are few works where conservative fields generated by a set of sinks, sources, vortices, dipoles and multipoles are tackled in an exact

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