

A communication approach to the superposition problem

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Abstract

In function theory the superposition problem is known as the problem of representing a continuous function $f(x_1, \dots, x_k)$ in k variables as the composition of "simpler" functions. This problem stems from the Hilbert's thirteenth problem. In computer science good formalization for the notion of composition of functions is formula. In the paper we consider real-valued continuous functions in k variables in the cube $[0,1]^k$ from the class \mathcal{H}^k_{ω} with ω a special modulus of continuity (measure the smoothness of a function) defined in the paper. \mathcal{H}^k_{ω} is a superset of Hölder class of functions. We present an explicit function $f \in \mathcal{H}^k_{\omega}$ which is hard in the sense that it cannot be represented in the following way as a formula: zero level (input) gates associated with variables $\{x_1, \dots, x_k\}$ (different input gates can be associated with the same variable $x_i \in \{x_1, \dots, x_k\}$), on the first level of the formula, arbitrary number $s \geq 1$ of t variable functions from \mathcal{H}^t_{ω} for $t < k$ are allowed, while the second (output) level may compute any s variable Hölder function. We apply communication complexity for constructing such hard explicit function. Notice that one can show the existence of such function using the "non constructive" proof method known in function theory as Kolmogorov's entropy method.

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Keywords

communication complexity, Hilbert 13th problem, superposition of continuous functions