Annals of Pure and Applied Logic 156 (2008) 13–20

Contents lists available at ScienceDirect

Annals of Pure and Applied Logic

journal homepage: www.elsevier.com/locate/apal

Structural properties of Q-degrees of n-c. e. sets

M.M. Arslanov [a,*], I.I. Batyrshin [a], R.Sh. Omanadze [b]

[a] Department of Mathematics, Kazan State University, Kazan, Russia

[b] Department of Pure Mathematics, Tbilisi State University, Tbilisi, Georgia

ARTICLE INFO

Article history:
Available online 2 July 2008

MSC:
03D20
03F30
03H15

Keywords:
Q-reducibility
Computably enumerable sets
Ershov hierarchy
Q-degrees

ABSTRACT

In this paper we study structural properties of n-c. e. Q-degrees. Two theorems contain results on the distribution of incomparable Q-degrees. In another theorem we prove that every incomplete Π^0_2 Q-degree forms a minimal pair in the c. e. degrees with a $\Delta^0_2 Q$-degree. In a further theorem it is proved that there exists a c. e. Q-degree $> \mathbf{0}$ that is not half of a minimal pair in the c. e. Q-degrees.

In this paper we study splitting and cupping–capping properties of n-c. e. Q-degrees. Recall [10] that a set A is Q-reducible to a set B if there is a computable function Φ such that for every $x \in \omega$, $x \in A \Leftrightarrow W_{\Phi(x)} \subseteq B$. In this case we say that $A \leq_Q B$ via Φ (or via a uniformly computably enumerable (c. e.) sequence of c. e. sets $U = \{U_x\}_{x\in\omega}$, if for all x $U_x = W_{\Phi(x)}$). If $A \leq_Q B$ via Φ we will also write $A = \Phi^B$, considering Φ as a Q-operator.

The relation of Q-reducibility is transitive and reflexive, so that it generates a degree structure on 2^ω. It is not hard to show that in general Q-reducibility is incomparable with Turing (T-)reducibility, but on c. e. sets if $A \leq_Q B$ then $A \leq_T B$. Therefore, on c. e. sets the relation $\leq_Q$ is strictly stronger than $\leq_T$, since $A \leq_Q B$ implies $\omega - A$ is B-c. e.

The interest in the study of the partial-ordering of Q-degrees increased after results of Dobritsa and Belegradek (see [3]). It follows from these results that every true sentence on the properties of Q-degrees in the language "$\leq$" becomes a true sentence on the relationship between classes of finitely generated subgroups of algebraically closed groups (for details see [4]).

In this paper we study some structural properties of Q-degrees of n-c. e. sets. A set A is n-c. e. for some $n \geq 1$ if there is a computable function $f(s, x)$ such that for every x:

$f(0, x) = 0$,

$A(x) = \lim_s f(s, x)$, and

$|\{s : f(s, x) \neq f(s+1, x)\}| \leq n$.

The 2-c. e. sets are also known as the d-c. e. sets as they are differences of c. e. sets.

A degree $\mathbf{a}$ is called an n-c. e. degree for $n \geq 1$ if it contains an n-c. e. set, and it is called a *properly* n-c. e. *degree* if it contains an n-c. e. set but no m-c. e. set for any $m < n$.

We adopt the usual notational conventions, found, for instance, in Soare [1987]. In particular, we write $[s]$ after functionals and formulas to indicate that every functional or parameter therein is evaluated at stage s. In particular, for an oracle X and c. e. functional Φ, $\Phi(X; y, s)$ means only that at most s steps are allowed for the computation from oracle X to converge;

* Corresponding author.
E-mail addresses: Marat.Arslanov@ksu.ru (M.M. Arslanov), ilnurb@yandex.ru (I.I. Batyrshin), r.omanadze@math.sci.tsu.ge (R.Sh. Omanadze).

0168-0072/$ – see front matter
doi:10.1016/j.apal.2008.06.003