Linear Algebra and its Applications 438 (2013) 71–79

Contents lists available at SciVerse ScienceDirect

Linear Algebra and its Applications

journal homepage: www.elsevier.com/locate/laa

Idempotents in a space with conjugation

Marjan Matvejchuk

Research Institute of Mathematics and Mechanics, ul. Universitetskaya 17, Kazan 420008, Russia

ARTICLE INFO

Article history:
Received 16 February 2011
Accepted 23 July 2012
Available online 13 September 2012

Submitted by H. Bart

AMS classification:
28A60
46C20
46C50
47A63
47B50

Keywords:
Hilbert space
Conjugation
Idempotent
Projection

ABSTRACT

Let H be a complex Hilbert space with conjugation operator J. We study J-real operators and we have covered J-regular subspaces. We prove that for given bounded idempotent p there exists a conjugation operator J_0 such that p is a J_0-projection, i.e. $p = J_0p^*J_0$.

1. Introduction

In [1, Chapter XII] the problem of construction of probability theory for quantum mechanics is posed. An analog of boolean algebra of events is quantum logic. An important interpretation of a quantum logic is the set $B(H)^{or}$ of all orthogonal (=self-adjoint) projections on a Hilbert space H. The problem to construct a quantum field theory sometimes leads to an indefinite metric space [2]. In the indefinite case, the set $B(H)^{Jo}$ of all J-orthogonal projections is an analog of the logics $B(H)^{or}$. It is important to know the properties and structure of projections in construction of measure theory on projections logic. J-projections were extensively studied at [3–6].

Let H be a complex Hilbert space with the Hilbert product $(\cdot,\cdot)$. Let $B(H)$ be the set of all bounded operators on H. Let us denote by $ran(A)$ the range of bounded operator $A \in B(H)$. Let J be a *conjugation* operator in H (see [7] $50), i.e. (1) $J^2 = I$, (2) $(Jx, Jy) = (y, x)$, for all $x, y \in H$. Note by (1), and (2),

E-mail address: Marjan.Matvejchuk@ksu.ru

0024-3795/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.laa.2012.08.003