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Coring stabilizers for a Hopf algebra coaction

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ABSTRACT

The coring stabilizer $\text{Stab}(P)$ is introduced for any prime ideal P of a right H -comodule algebra A such that the factor ring A/P is either right or left Goldie. This notion is used to obtain Hopf algebraic analogs of two category equivalences associated with a homogeneous space. The category of linearized quasicoherent sheaves on a noncommutative homogeneous space is interpreted as a suitable quotient category of the category of Hopf modules. Birational invariance of such quotient categories is proved. It is shown that for a birational H -coequivariant extension B of A properly defined subsets of prime ideals of A and B correspond to each other bijectively.

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Introduction

Let H be a Hopf algebra and A a right H -comodule algebra. Suppose that P is a prime ideal of A such that the factor ring A/P is either right or left Goldie, so that A/P has a simple artinian classical right or left quotient ring Q_P . We will introduce two Q_P -corings $\text{Stab}(P)$ and $\text{Inert}(P)$ called the *stabilizer* and the *inertializer* of P . The latter is a factor coring of the former. When A is commutative and H is the function algebra on a finite group, these corings are described explicitly in terms of the set-theoretic stabilizer and the inertia group of P with respect to the group action corresponding to the coaction of H on A . When A is commutative and H is an arbitrary commutative Hopf algebra, $\text{Inert}(P)$ coincides with the Hopf algebra over the field Q_P representing the scheme-theoretic stabilizer (inertia group) of P in the group scheme represented by H . It seems that in the noncommutative case too much of information about the original coaction of H may get lost in $\text{Inert}(P)$, and we will not use the inertializers in any way.

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