

# New results on the bijectivity of antipode of a Hopf algebra

Serge Skryabin

*Chebotarev Research Institute, Universitetskaya St. 17, 420008 Kazan, Russia*

Received 18 November 2005

Available online 5 June 2006

Communicated by Susan Montgomery

---

## Abstract

Two results giving sufficient conditions for the bijectivity of the antipode of a Hopf algebra are proved.  
© 2006 Published by Elsevier Inc.

---

## 0. Introduction

Let  $H$  be a Hopf algebra over a field  $k$  and  $s : H \rightarrow H$  its antipode. A classical result due to Larson and Sweedler [4] says that  $s$  is bijective provided that  $\dim H < \infty$ . As was established by Radford [7], the same conclusion holds under the assumption that  $H^*$  contains nonzero integrals (that is,  $H$  is co-Frobenius). On the other hand, Takeuchi [13] constructed the free Hopf algebra  $H(C)$  on a coalgebra  $C$  whose antipode is not bijective when  $C$  is the matrix coalgebra  $\text{Mat}_n(k)^*$ , that is, the dual of the matrix algebra  $\text{Mat}_n(k)$ , with  $n > 1$ . One might expect that the bijectivity of  $s$  depends on some finiteness conditions. The next result proved in the present paper uses purely ring-theoretic restrictions on  $H$ :

### Theorem A.

- (i) *If  $H$  is weakly finite then  $s$  is injective.*
- (ii) *If  $H$  can be embedded into a left perfect ring  $Q$  such that  $Q$  is an essential extension of  $H$  as a right  $H$ -module, then  $s$  is bijective.*

---

*E-mail addresses:* [sskryabi@vub.ac.be](mailto:sskryabi@vub.ac.be), [serge.skryabin@ksu.ru](mailto:serge.skryabin@ksu.ru).