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Iterative methods and parallel solution of the dam problem

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Abstract

A free boundary problem of fluid flow in the porous medium is considered. A finite difference scheme is constructed and written in the form of the inclusion for the sum of several multi-valued operators. The study of this mesh scheme and iterative algorithms of its numerical solution is performed. Solution of the mesh problem by Gauss–Seidel and by Schwarz alternating methods was executed. The numerical results are reported which confirm the theoretical results.

Keywords: Dam problem; Domain decomposition methods; Parallel solution

1. Introduction

The first variational formulation of the dam problem was given in Baiocchi's article [6] where the dam of the rectangular form was considered. A number of articles concerned this class of dam problem (cf. [7,8], survey [11] and the bibliography of this survey).

A new mathematical model for the filtration problem in the dam of arbitrary geometry was constructed in [1,10]. The existence of a weak solution was proved. The uniqueness problem was solved in [13,14]. The properties of the free boundary were studied in [3]. The main results in this field can be found in the monograph [16], cf. also [12].

Some more recent articles deal with the weak solutions of the linear and non-linear seepage problems in the bounded and unbounded domains (cf. [15,22,23] and the bibliography therein).

The mesh schemes for the dam problem with arbitrary geometry of the dam were first studied in [2] and then in [9,24]. The mixed formulation of free boundary problems of fluid flow and mixed finite element methods in its approximation were proposed and studied in [5].

Parallel solutions of the dam problem are discussed in [17,25], where the mesh scheme for Baiocchi's model is considered.

In this article we study the problem of water filtration through and under the dam with the permeable foundation and with the immovable fluid under the dam of greater density than in the moving water. These liquids are supposed to be immiscible. In this case the unknown domain of the filtration has two free boundaries, namely, the depression surface which separates the wet and the dry parts of the dam and the interface between the two fluids.

The mathematical model of the problem that we consider here was formulated and studied in [21]. The existence of the unique weak solution was proved, and the weak convergence of a sequence of

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