

A mean convergence theorem and weak law for arrays of random elements in martingale type p Banach spaces

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Abstract

For weighted sums of the form $S_n = \sum_{j=1}^{k_n} a_{nj}(V_{nj} - C_{nj})$ where $\{a_{nj}, 1 \leq j \leq k_n < \infty, n \geq 1\}$ are constants, $\{V_{nj}, 1 \leq j \leq k_n, n \geq 1\}$ are random elements in a real separable martingale type p Banach space, and $\{C_{nj}, 1 \leq j \leq k_n, n \geq 1\}$ are suitable conditional expectations, a mean convergence theorem and a general weak law of large numbers are established. These results take the form $\|S_n\| \rightarrow_{\mathcal{L}^r} 0$ and $S_n \rightarrow_P 0$, respectively. No conditions are imposed on the joint distributions of the $\{V_{nj}, 1 \leq j \leq k_n, n \geq 1\}$. The mean convergence theorem is proved assuming that $\{\|V_{nj}\|_r, 1 \leq j \leq k_n, n \geq 1\}$ is $\{|a_{nj}|_r\}$ -uniformly integrable whereas the weak law is proved under a Cesàro type condition which is weaker than Cesàro uniform integrability. The sharpness of the results is illustrated by an example. The current work extends that of Gut (1992) and Hong and Oh (1995).

Keywords

Array of random elements, Cesàro uniformly integrable array, Convergence in \mathcal{L}^r , Convergence in probability, Real separable martingale type p Banach space, Weak law of large numbers, Weighted sums, $\{a_{nj}\}$ -uniformly integrable array