Abstract

A theoretical investigation of nonlinear steady and unsteady roll waves on shallow layers of non-Newtonian dilatant fluids considered in the frame of the two-parametric power-law model of Ostwald–de Waele and moving down inclines is presented. For the profiles of steady waves an ordinary differential equation is derived, and its analysis shows that smoothed hydraulic jumps exist when their amplitudes are smaller than a certain critical value of a variable which depends on the fluid index and the Ostwald–de Waele number. Numerical solutions to the problem of propagation of unsteady waves are obtained.

Keywords: Non-Newtonian fluids; Shallow layers; Hydraulic jumps; Steady; Unsteady waves

1. Introduction

Many natural and technological processes are associated with flows of fluids, the viscosity of which depends on the shear rate. Such non-Newtonian fluids (for example, mud flows, volcanic lava, polymeric solutions, oil, honey, grainy materials) can be adequately described by the Ostwald–de Waele two-parametric power-law model. The first parameter is the viscosity coefficient η of dimension (kg m^{-1} s^{n-2}), the second is the flow index n ($0 < n < 1$ for a shear-thinning or pseudoplastic fluid and $n > 1$ for a shear-thickening or dilatant fluid). Mud flows, for which $0.1 \le n \le 0.4$ and oil with $n = 0.8$, represent pseudoplastic liquids, but lime-water mixture with $n = 1.5$ and honey, for which $n = 2.5$, are dilatant liquids [1]. The special case of $n = 1$ corresponds to a Newtonian fluid. Note that a dry granular material in regimes characterized by large solid concentrations and low to moderate shear rates from 1 to 10 s^{-1} can be considered as a non-Newtonian dilatant medium with index $n = 2$.

The exhaustive analysis of instability of incompressible non-Newtonian power-law thin liquid layers of a constant undisturbed depth at an inclination angle α, described by the depth-averaged balance laws with respect to infinitesimal periodic disturbances is represented in [2,3] without considering surface

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