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Schwarz-pick inequalities for derivatives of arbitrary order

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Abstract

Let Ω and Π be two simply connected domains in the complex plane C which are not equal to the whole plane C and let λ_Ω and λ_Π denote the densities of the Poincaré metric in Ω and Π, respectively. For f : Ω → Π analytic in Ω, inequalities of the type $|f^{(n)}(z)|/n! \leq M_n(z, \Omega, \Pi)(\lambda_\Omega(z))^n/\lambda_\Pi(f(z))$, $z \in \Omega$, are considered where $M_n(z, \Omega, \Pi)$ does not depend on f and represents the smallest value possible at this place. We prove that $M_n(z, \Delta, \Pi) = (1 + |z|)^{n-1}$ if Δ is the unit disk and Π is a convex domain. This generalizes a result of St. Ruscheweyh. Furthermore, we show that $C_n(\Omega, \Pi) = \sup\{M_n(z, \Omega, \Pi) \mid z \in \Omega\} \leq 4^{n-1}$ holds for arbitrary simply connected domains whereas the inequality $2^{n-1} \leq C_n(\Omega, \Pi)$ is proved only under some technical restrictions upon Ω and Π.

Keywords

Density of the Poincaré metric, Derivatives of arbitrary order, Schwarz-Pick lemma