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The minimal e-degree problem in fragments of Peano arithmetic

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Abstract

We study the minimal enumeration degree (e-degree) problem in models of fragments of Peano arithmetic (*PA*) and prove the following results: in any model $\mathcal{M}$ of Σ_2 induction, there is a minimal enumeration degree if and only if $\mathcal{M}$ is a nonstandard model. Furthermore, any cut in such a model has minimal e-degree. By contrast, this phenomenon fails in the absence of Σ_2 induction. In fact, whether every Σ_2 cut has minimal e-degree is independent of the Σ_2 bounding principle.

MSC: 03D20; 03F30; 03H15

1. Introduction

This paper is motivated by the study of Turing degrees in fragments of Peano arithmetic (*PA*), a subject which was developed in the 1980's. We recommend Chong and Yang [4,5] for the basic notions and results, as well as some open problems in this subject. The study

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