Annals of Pure and Applied Logic 1996 vol.78 N1-3, pages 29-56

Interpolating d-r.e. and REA degrees between r.e. degrees

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Abstract

We provide three new results about interpolating 2-r.e. (i.e. d-r.e.) or 2-REA (recursively enumerable in and above) degrees between given r.e. degrees: Proposition 1.13. If c < h are r.e., c is low and h is high, then there is an a < h which is REA in c but not r.e. Theorem 2.1. For all high r.e. degrees h < g there is a properly d-r.e. degree a such that h < a < g and a is r.e. in h. Theorem 3.1. There is an incomplete nonrecursive r.e. A such that every set REA in A and recursive in 0′ is of r.e. degree. The first proof is a variation on the construction of Soare and Stob (1982). The second combines highness with a modified version of the proof strategy of Cooper et al. (1989). The third theorem is a rather surprising result with a somewhat unusual proof strategy. Its proof is a 0‴ argument that at times moves left in the tree so that the accessible nodes are not linearly ordered at each stage. Thus the construction lacks a true path in the usual sense. Two substitute notions fill this role: The true nodes are the leftmost ones accessible infinitely often; the semitrue nodes are the leftmost ones such that there are infinitely many stages at which some extension is accessible. Another unusual feature of the construction is that it involves using distinct priority orderings to control the interactions of different parts of the construction.