

Asymptotically sharp bounds in the Hardy-Littlewood inequalities on mean values of analytic functions

Avkhadiiev F., Wirths K.

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

Let f be analytic in the unit disc, and let it belong to the Hardy space H^p , equipped with the usual norm $\|f\|_p$. It is known from the work of Hardy and Littlewood that for $q > p$, the constants $C(p, q) := \sup\{ \int_0^1 \int_0^{2\pi} (1-r)^{-p/q} |f(re^{i\theta})|^q d\theta \}^{1/q} / \|f\|_p = 1$, with the usual extension to the case where $q = \infty$, have $C(p, q) < \infty$. The authors prove that $\lim_{q \rightarrow p^+} C(p, q) = 1$, \inf_p