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Toral Rank One Simple Lie Algebras of Low Characteristics

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INTRODUCTION

Recent years have seen a successful completion of the classification of simple finite dimensional Lie algebras over algebraically closed fields of characteristic $p > 7$. The work on characteristics $5, 7$ is in progress now. Much more difficult are cases of characteristics $2, 3$ which display an abundance of simple Lie algebras that fit into neither of the two major classes: classical and Cartan type algebras (see [19] where most of the known examples in characteristic 3 are discussed). Only a few classification results under very restrictive hypotheses have been known.

The general classification scheme developed by R. E. Block and R. L. Wilson in case of restricted Lie algebras [3] and then extended by H. Strade to the nonrestricted case [23] is based on the study of root space decompositions $\sum L_\alpha$ of a simple Lie algebra L with respect to tori T of maximal dimension in the minimal p-envelope of L. The dimension of those tori is called the (absolute) toral rank of L [22]. With each subgroup $\Gamma \subset T^*$ generated by r roots linearly independent over the prime field one associates a rank r section $\sum_{\alpha \in \Gamma} L_\alpha \subset L$ and tries to glue together information about sections of low rank and their cores, the factor algebras by solvable radicals. As a first step, one has to determine the cores of rank one sections. That leads to the necessity to know toral rank one simple Lie algebras. In the present paper we solve this problem for $p = 2, 3$.

The main result, Theorem 6.5, states that only simple Lie algebras sl(2) and psl(3), i.e., the factor algebra of sl(3) by its one-dimensional center,

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