PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 4097–4106

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Contact melting inside an elastic capsule

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Received 28 August 2001; received in revised form 26 March 2002

Abstract

An approximate mathematical model of contact melting of an unfixed material in elastic cylindrical and spherical capsules is developed. Since the density of the solid is higher than that of the melt, the melting solid resides at the bottom supported by a thin layer of the generated, convecting, melt, and the capsule swells. The main characteristic scales and non-dimensional parameters, which describe the principal features of the melting process and the liquid flow, are found. Linearisation with regard to the Stefan number as well as the small difference between the densities of the solid and liquid enables us to derive a closed-form evolution equation for the motion of the solid, which also determines the melting rate. Numerical solution of the evolution equation shows that the swelling of the capsule during melting, which is caused by the decrease of the density during phase transition, leads to slowing down of the melting process. This effect is due to flattening of the lower surface of the capsule, which entails fall of the pressure along with thickening of the molten layer. The latter determines the decrease of the melting rate. © 2002 Published by Elsevier Science Ltd.

1. Introduction

Analysis of close-contact melting of a solid in a cavity is motivated by application in latent heat-of-fusion thermal-storage systems. Contact melting in a circular horizontal cylinder has been studied numerically by Saitoh and Hirose [1] and Nicholas and Bayazitoglu [2], where the energy equation was solved in a thin molten layer, which determined the sinking velocity of the solid core. Analytical solution was found by Bareiss and Beer [3], who showed its good agreement with experimental results for small Stefan numbers. Contact melting in a spherical capsule was investigated numerically by Moore and Bayazitoglu [4] and later, applying the technique proposed in [3], Bahrami and Wang [5], Roy and Sengupta [6] as well as Fomin and Saitoh [7] reported analytical solutions. Fomin et al. [8] studied the effect of the shape factor of elongated capsules on the melting rate. The general scheme for the scale analysis of the contact melting problem was proposed by Bejan [9]. Although the aforementioned investigations highlight the main characteristics of contact melting inside a capsule, the effect of the difference between the densities of the solid and liquid phase, which leads to swelling of the capsule, has not been yet analysed. In the present paper the approximate approach developed by Bareiss and Beer [3] is applied with the higher order of accuracy with regard to non-linear temperature distribution in the melt, accounting for the convective heat transfer, for the mathematical modelling of contact melting in elastic horizontal circular cylinder and sphere. The main objective of the work is to derive a closed-form evolution equation for the solid motion and estimate the effect of the capsule swelling on the melting rate.

2. System model and analysis

Melting process within a circular capsule is illustrated in Fig. 1. The equation $(x^*/a)^2 + (y^*/a)^2 = 1$ describes the generating curve of the internal surface of the capsule which can be a horizontal circular cylinder or a sphere completely filled with solid phase. In the first case (x^*, y^*) are Cartesian coordinates, and in the second case

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0017-9310/02/$ - see front matter © 2002 Published by Elsevier Science Ltd.
PII: S0017-9310(02)00121-7