PARTICLE CREATION BY A BLACK HOLE AS A CONSEQUENCE OF THE CASIMIR EFFECT

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The accelerated-mirror results are applied to a black hole. As a consequence the hole produces black-body radiation. Its temperature exactly coincides with Hawking's result. An important difference between the energy–momentum tensors of scalar and electromagnetic fields near the horizon is discovered.

In a previous paper [1] a programme of investigation of black-hole evaporation with the help of the Casimir effect in Minkowski spacetime was proposed. As is well known, the gravitational field of a black hole acts as a barrier for propagation of massless waves. The consideration of the peak of the barrier ($r = 3M$; $c = G = 1$) as the surface of a reflecting sphere permitted us to apply the results of various calculations of the Casimir effect to a black hole. It appeared that the flow of negative Casimir energy should cause the area of the horizon to shrink at a rate consistent with the energy flux observed at infinity. The rough estimate based on the assumption about the black-body nature of radiation showed that its temperature should be inversely proportional to hole's mass M. The purpose of this paper is to demonstrate that the only existence of the horizon and of the potential barrier is sufficient to compel the hole to emit black-body radiation with a temperature that exactly coincides with the result of ref. [2].

A. Consider a particle which is at rest in the gravitational field of a nonrotating black hole. Its four-velocity is

$$u^\alpha \equiv dx^\alpha/d\tau = ((1-2M/r)^{-1/2}, 0, 0, 0)\,.$$

The proper acceleration of the particle is

$$a^\alpha \equiv \mathcal{D}u^\alpha/d\tau = du^\alpha/d\tau + \Gamma^\alpha_{\beta\gamma}u^\beta u^\gamma = \Gamma^\alpha_{tt}u^t u^t$$

($\alpha, \beta, \gamma = t, r, \theta, \varphi$). The only nonvanishing component of Γ^α_{tt} is $\Gamma^r_{tt} = (M/r^2)(1-2M/r)$. Hence $a^\alpha = (0, M/r^2, 0, 0)$,

$$|a| \equiv (g_{\alpha\beta}a^\alpha a^\beta)^{1/2} = (1-2M/r)^{-1/2}M/r^2\,. \tag{1}$$

A stationary distant observer will measure

$$d^\alpha = (\mathcal{D}u^\alpha/d\tau)d\tau/dt = a^\alpha(1-2M/r)^{1/2}\,;$$

$$|d| \equiv (g_{\alpha\beta}d^\alpha d^\beta)^{1/2} = M/r^2\,. \tag{2}$$

The potential barrier (localized in the vicinity of $r = 3M$) has a nonzero proper acceleration $b^{-1} \cong (3\sqrt{3}M)^{-1}$. The goal of this paper can be achieved by application of the accelerated-mirror results.

B. The stresses induced in the Minkowski vacuum by an infinite plane conductor that is uniformly accelerated normal to itself were investigated by Candelas and Deutsch [3]. The solution of the boundary problem was facilitated by introduction of accelerated (Rindler) coordinates ξ and τ.

$$t = \xi \operatorname{sh}\tau\,, \quad x = \xi \operatorname{ch}\tau\,,$$

$$ds^2 = \xi^2 d\tau^2 + d\xi^2 + dy^2 + dz^2\,.$$

In this system the curves ξ = const., y = const., z = const. are worldlines of constant proper acceleration ξ^{-1}. The surface $\xi = b$ = const. represents the trajectory of the barrier.

Candelas and Deutsch calculated the regularized vacuum expectation value $\langle T^\mu_\nu\rangle = \langle 0|T^\mu_\nu|0\rangle$ of the stress energy tensor far from the conductor ($\xi/b \to \infty$). For a scalar field