

AN ALGORITHM FOR THE DIFFERENTIATION OF ARBITRARY ORDER IN THE SOLUTION OF INVERSION PROBLEMS IN APPLIED SPECTROSCOPY

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Abstract—We describe algorithms for the solution of inversion problems in atomic absorption and resonance fluorescence. Also derived is the thin-layer approximation for solution of Abel's equation, with or without absorption. The basis for these procedures is the algorithm of differentiation to arbitrary order, including fractional differentiation. Mathematical experiments demonstrating the efficiency of the suggested algorithm have been carried out.

INTRODUCTION

One is frequently confronted by the necessity to solve inversion problems in various fields of applied spectroscopy. Allowance for equipment distortions of the spectrum necessitates inversion to an ideal spectral device. Allowance for the transverse vibrations of a plasma column is bound with reduction to its fixed position in space and time. In a non-homogeneous plasma, reduction to the thin layer, i.e., Radon's or Abel's inversion, is also necessary. The authors of Refs. 1–6 consider algorithms based on the statistical regularization method and used for the solution of inversion problems in plasma spectroscopy. In this paper, we consider algorithms for solving inversion problems in atomic absorption and resonance fluorescence and also the problems of inversion to the thin-layer in solving Abel's equation with a constant absorption coefficient and Abel's generalized equation in the absence of absorption. A common feature of these problems is the fact that in their solution an efficient regularized algorithm of differentiation of arbitrary order q (including fractional values) can be effectively used.

PROBLEM FORMULATION

Restoration of the source function in atomic absorption and resonance-fluorescence problems

Any receiving and recording equipment possesses inertia, which results in signal distortion.⁷ The effect of the measuring circuit on a signal is similar to introduction of an RC-circuit with a capacitor at the output. In this case, the unit-step function of the recording circuit is expressed in the form

$$U(t) = 1 - \exp(-t/\tau), \quad (1)$$

where τ is the time constant of the recording unit. The signal at the output of the receiving and recording unit $F(t)$ is described by the following function:

$$F(t) = \int_0^t A(t')R(t-t') dt', \quad (2)$$

where $R(t)$ is the temporal derivative of $U(t)$. The restoration of $A(t)$ from Eq. (2) is an inversion problem, which is solved using Volterra's integral equation of the first kind. The absorption signal $A(t)$ is distorted by the recording unit and is described by Eq. (2). The number of atoms $N(t)$ found in the analysis zone at any given moment is given by⁸

$$N(t) = \int_0^t S(t')R_a(t-t') dt', \quad (3)$$