

UDC 514.753.2

ON THE 3-TRANSITIVITY OF TRANSFORMATION GROUP OF THE LOBACHEVSKY PLANE IN THE BELTRAMI-KLEIN MODEL

L.I. Nigmatullina¹¹ *leysan.nigmatullina@bk.ru*; Kazan (Volga) federal university

In this paper the transformation group of the Lobachevsky plane in the Beltrami-Klein model that extends the group of motions is considered. One property of this group analogous to the 3-transitivity is established.

Keywords: Lobachevsky space, metric, transformation group.

Let $\Lambda = B(O, 1)$ be an open disk in a Euclidean plane of radius 1 with the center at a fixed point O . Points inside the circle will be given by their radius vectors relative to the point O . The l-point is a point in Λ , and the l-line is a chord in Λ . Distance between l-points $x, y \in \Lambda$ is calculated by the formula

$$\rho(x, y) = k \operatorname{Arch} \frac{1 - (x, y)}{\sqrt{1 - x^2} \sqrt{1 - y^2}},$$

where k is a positive constant, (x, y) is the scalar product of the radius vectors x, y and x^2 is the scalar square of the radius vector x [1]. The pair (Λ, ρ) is called the Beltrami-Klein model of the Lobachevsky plane. Consider $\lambda \in \mathbb{R}$, $p, x \in \Lambda$. We define the point $\lambda_p(x) \in \Lambda$ by the following three conditions [1, 2, 3]

1.

$$\rho(p, \lambda_p(x)) = |\lambda| \rho(p, x).$$

2. For $x \neq p$, $\lambda > 0$, the points $x, \lambda_p(x)$ lie on the l-line $P(p, x)$ passing through the points p, x , on one side of the point p .

3. For $x \neq p$, $\lambda < 0$, the points $x, \lambda_p(x)$ lie on the l-line $P(p, x)$ passing through the points p, x , on the opposite sides of the point p .

The explicit formula for $\lambda_p(x)$ has the following form [1, 3]

$$\lambda_p(x) = \frac{p \operatorname{ch}(a) \operatorname{sh}((1 - \lambda)c) + x \operatorname{ch}(b) \operatorname{sh}(\lambda c)}{\operatorname{ch}(a) \operatorname{sh}((1 - \lambda)c) + \operatorname{ch}(b) \operatorname{sh}(\lambda c)},$$

where $a = \rho(0, p)/k$, $b = \rho(0, x)/k$, $c = \rho(p, x)/k$.

The parallel transfer $g_a : \Lambda \rightarrow \Lambda$ along the axis Ox onto the vector a can be represented as

$$\hat{x} = \frac{x + p}{1 + px}, \quad \hat{y} = \frac{y \sqrt{1 - p^2}}{1 + px},$$

where $p = |a|$. In the Beltrami-Klein model, an arbitrary motion has the form $f = g_a \circ U$, where U is an orthogonal transformation of the Euclidean plane with a fixed point O , bounded by the disk Λ [1]. We introduce the group G of transformations in the Beltrami-Klein model of the Lobachevsky plane. An arbitrary element of G is a composition of a finite number of transformations of the following three kinds:

1) the rotations around the point O with a possible reflection with respect to axis Ox ;

- 2) the parallel translations g_a , where $a \in \mathbb{R}^2$ and $0 \leq |a| < 1$;
 3) the transformations $\lambda_0: \Lambda \rightarrow \Lambda$, $\lambda_0(x) = x \operatorname{cth} b \operatorname{th}(\lambda b)$, where $\lambda \in \mathbb{R}$.
 The following proposition is the main result of the paper.

Statement. *Let $\{A, B, C\}, \{A', B', C'\}$ be two triples of pairwise distinct points of the plane Λ . Then for sufficiently small $\varepsilon > 0$, applying the Maxima program, we find an element $g \in G$ such that $g(A) \in B(A', \varepsilon)$, $g(B) \in B(B', \varepsilon)$, $g(C) \in B(C', \varepsilon)$.*

Comment. 1. *We wrote the algorithm and the program in the built-in Maxima language in order to calculate the element $g \in G$ described in the statement.*

2. *This statement allows us to assume that the group G acts on the Lobachevsky plane in the Beltrami–Klein model 3-transitive.*

References

1. Sosov E. N. *The Lobachevsky geometry and its application in the special theory of relativity: teaching manual*. – Kazan: Kazan university, 2016. – 84 p.
2. Sabinin L. V. *Odules as a new approach to a geometry with a connection* // Soviet Math. Dokl., 1977. – V. 233. – N. 5. – P. 800–803.
3. Sosov E. N. *On the action of the multiplicative group of nonzero real numbers on the pointed Lobachevsky space* // Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki, 2012. – V. 154. – N. 4. – P. 156–160.

О 3-ТРАНЗИТИВНОСТИ ГРУППЫ ПРЕОБРАЗОВАНИЙ ПЛОСКОСТИ ЛОБАЧЕВСКОГО В МОДЕЛИ БЕЛЬТРАМИ–КЛЕЙНА

Л.И. Нигматуллина

В статье в модели Бельтрами–Клейна плоскости Лобачевского рассматривается группа преобразований, которая является расширением группы движений. Устанавливается свойство этой группы, аналогичное свойству 3-транзитивности.

Ключевые слова: плоскость Лобачевского, метрика, группа преобразований.

UDC 514.822

WORMHOLES

A.A. Popov¹

¹ apopov@kpfu.ru; Kazan Federal University

In this paper we give an overview of wormhole solutions in the framework of the general theory of relativity.

Keywords: wormhole, theory of gravity, self-consistent solution.

By definition a wormhole is a bridge connecting two asymptotically flat regions. Usually such construction is a classical object and should satisfy to Einstein equations. The topology of the 4D wormholes is the topology of direct product of the Minkowski plane and a unit sphere. Static traversable wormholes could be threaded by "exotic matter" that violates certain energy conditions at least at the throat [1]. As an example of such a matter