

# Thermal effect in the Casimir force for graphene and graphene-coated substrates: Impact of nonzero mass gap and chemical potential

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## Abstract

© 2017 American Physical Society. The rigorous finite-temperature QED formalism of the polarization tensor is used to study the combined effect of nonzero mass gap  $m$  and chemical potential  $\mu$  on the Casimir force and its thermal correction in the experimentally relevant configuration of a Au sphere interacting with a real graphene sheet or with graphene-coated dielectric substrates made of different materials. It is shown that for both a free-standing graphene sheet and for graphene-coated substrates the magnitude of the Casimir force decreases as  $m$  is increased, while it increases as  $\mu$  is increased, indicating that these parameters act in opposite directions. According to our results, the impact of  $m$  and/or  $\mu$  on the Casimir force for graphene-coated plates is much smaller than for a free-standing graphene sheet. Furthermore, computations show that the Casimir force is much stronger for graphene-coated substrates than for a free-standing graphene sample, but the thermal correction and its fractional weight in the total force are smaller in the former case. These results are applied to a differential setup that was recently proposed to observe the giant thermal effect in the Casimir force for graphene. We show that this experiment remains feasible even after taking into account the influence of the nonzero mass-gap and chemical potential of real graphene samples. Possible further applications of the obtained results are discussed.

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