

# Arrows of times, non-integer operators, self-similar structures, zeta functions and Riemann hypothesis: A synthetic categorical approach

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## Abstract

© 2017 L & H Scientific Publishing, LLC. The authors have previously reported the existence of a morphism between the Riemann zeta function and the "Cole and Cole" canonical transfer functions observed in dielectric relaxation, electrochemistry, mechanics and electromagnetism. The link with self-similar structures has been addressed for a long time and likewise the discovered of the incompleteness which may be attached to any dynamics controlled by non-integer derivative operators. Furthermore it was already shown that the Riemann Hypothesis can be associated with a transition of an order parameter given by the geometric phase attached to the fractional operators. The aim of this note is to show that all these properties have a generic basis in category theory. The highlighting of the incompleteness of non-integer operators considered as critical by some authors is relevant, but the use of the morphism with zeta function reduces the operational impact of this issue without limited its epistemological consequences.

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## Keywords

Fractal geometry, Non integer operators, Riemann hypothesis, Zeta function

## References

- [1] Badiali, J.P. (2005), Entropy, time irreversibility and Schrodinger equation in a primary discrete space, Journal of Physics, A. 38, 2835-2847.
- [2] Badiali, J.P. (2015), Toward the new foundation of statistical thermodynamics, arXiv 1311. 4995, [Cond. Mat. Stat. Mech]
- [3] Badiali, J.P. (2015), The role of paths integral approach in statistical thermodynamics, arXiv: 1503.04122, [Cond. Mat. Stat. Mech].
- [4] Barnett, J. and Smolin, L. (2015), Fermion doubling in loop quantum gravity, arXiv: 1507-01232 [Gr-Qc].
- [5] Penrose, R. (2010), Cycles of time. An extraordinary new view of the universe, Vintage Books: New York.
- [6] Petitot, J. (2015), Non Commutative Geometry and Physics (February 2007), arXiv: 1505-001132, [Maths- Ph]
- [7] Connes, A. and Marcolli, M. (2008), Non Commutative Geometry, Quantum Fields and Motives, AMS Colloquium Vol 55
- [8] Rovelli, C. (2015), Par-delà le visible: La Réalité du monde physique et la Gravité quantique, Odile Jacob Editions: Paris. See also: <http://www.theorie.physik.uni-goettingen.de/forschung/qft/theses/dipl/Paetz.pdf>
- [9] Leinster, T. (2014), Basic Category theory, Cambridge University Press: Cambridge.

- [10] Coecker, B. (2011), *New structures for physics*, Lecture note in physics, 813, Springer Verlag: Berlin.
- [11] Le Mehaute, A. (1991), *Fractal Geometry and applications*, Penton Press (from French Hermes edition 1990): London.
- [12] Le Mehaute, A., Nigmatullin, R., and Nivanen, L. (1997), *Fleches du temps et geometries fractales*, Hermes Edition: Paris
- [13] Jonsher, A.K. (1983), *Dielectric relaxation in solids*, Chelsea Dielectric Press: London.
- [14] Ellenbroek, W.G., Hagh, V.F., Kumar A., Thorpe M.F., and van Hecke M. (2015), Rigidity loss in disordered systems: Three scenarios, *Physical Review Letters*, 114, 135501.
- [15] Le Mehaute, A., Heliodore F., and Cottevieille D. (1992), Ondes en milieu fractal, *Revue scientifique et technique de la defense*, 92, 23-33.
- [16] Le Mehaute, A., El Kaabouchi, A., and Nivanen L. (2010), Riemann Conjecture and Fractional derivative, *Computer & Mathematics with Applications*, 59 (5) 1610-1613.
- [17] Sabatier, J. (2015), Quo Vadimus point of view of Jocelyn Sabatier, *Fractional Calculus and applied analysis*, 18 (2), 495-526.
- [18] Liu, S.H.,(1985), Fractal model for ac response of rough interface, *Physical review letters*, 55, 529-535.
- [19] Sabatier, J., Merveillaut, M., Malti, R, and Oustaloup, A. (2008), On the representation of Fractional Order Systems: interest for intial condition problem, 3. IFAC Workshop on Fractional differentiation and applications Ankara Turkey Nov 5-7.
- [20] Sabatier, J., Merveillaut, M., Malti, R. and Oustaloup, A., (2010), How to impose physically coherence initial conditions to a fractional system?, *Communication in nonlinear science and numerical simulation*, 15 (5), 1318-1326.
- [21] Sabatier, J. and Frages, C., (2014), Long memory model: a first solution to the infinite energy storage ability of linear time-invariant fractional models, 19 World Congress of International Federation of Automatic control, Cape Town, South Africa, Aug 24-29.
- [22] Sabatier, J., Frages, C. and Trigeassou, J.C., (2014), Fractional systems state space description: some wrong ideas and proposed solutions, *Journal on vibration and control*, 20 (7), 1076-1084.
- [23] Keating, J.P. and Snaith, C.N., (2000) Random matrix theory and zeta function. *Comm. Math. Phys* 57-89.
- [24] Hines, P. (1999), The categorical theory of self similarity, *Theoretical and Applied Categories*, 6, 33-46.
- [25] Wolf M. (2015), Will physicist prove the Riemann Hypothesis, arXiv:1410-1214 [Math-Ph].
- [26] Riot, P. and le Mehaute, A. (2016), A categorical approach of Riemann Hypothesis, *Computer & Mathematics with Applications* (Submitted).
- [27] Markl, M., Shnidder, S., and Stasheff, J. (2002), *Operad in algebra, topology and physics*, American Mathematical Society, *Mathematical Survey and Monographs*, 14.
- [28] Lesne, A. (1999), *Methodes de renormalisation: Phénomènes Critiques*. Chaos. Structures fractales, Edition Eyrolles: Paris.
- [29] Oustaloup, A. (1981), The fractional order sinusoidal oscillators: optimization and their uses in highly linear FM modulation, *IEEE transaction* 28(10) 1007-1009.
- [30] Lambek, J. (1968), A fix point theorem for complete categories, *Mathematische Zeitschrift*, 103(2), 151-161.
- [31] Scott, D.S. (1975), Data types as lattices in *Proceeding of international summer institute and logic colloquium.*, *Lectures notes in mathematics*, 499, 579-651, Springer Verlag: Berlin.
- [32] Smyth, M.B. and Plotkin, G. (1982), The category theory solution of recursive equations, *SIAM Journal of Computation*, 11(4), 761-783.
- [33] Le Mehaute, A., De Guibert, A., Delaye M, and Fillipi, C. (1982), Transfert d'energie sur interface a similitude interne, *Comptes Rendus de l'Académie des Sciences de Paris* 294 (II), 865-868.
- [34] Podlubny, Y. (1999), *Fractional differential equations*, Academic Press: Londre.
- [35] Hines, P. (2003), A categorical framework for finite state machines, *Math. Struct. Comp.*, 13(3), 451-480.
- [36] Hines, P. (2008), From causality to computational models, *International Journal. of Unconventional Computation.*, 4(3), 249-272.
- [37] Voronin, S. (1975), Theorem on the Universality of the Riemann Zeta Function. *Izv. Akad. Nauk SSSR, Ser. Matem.* 39, 475-486.
- [38] Reprinted in *Math USSR Izv.* 9, 443-445.
- [39] Belaïche, A. (1996), The tangent space in sets Riemannian geometry, in *Singular trajectories and their control theory*, in *Progress in Mathematics*, 4, 77, A. Belaïche and Risler J.S. Eds, Birkhauser Verlag: Berlin.
- [40] Bagchi, G. (1982), Join universality theorem for Diricklet L function, *Maths. Z.*, 181, 319-334.
- [41] Le Mehaute, A., Nivanen, L., and Nigmatulli, R. (2005), Space time with bounded curvature and non-integral differential equation, in *Fractional Differentials and its applications*, A. le Mehaute, J.P.T. Machado, J.C. Trigeassou and J. Sabatier Eds. U-Book on demand, (ISBN 3-86608-026-3) 231-262.

- [42] Riot, Kahlem, L. and Le Mehaute, A. (1996), Les fondements geometriques et algebrique de la fractalite, Annales des telecommunications, 51 (9), 567-572.
- [43] Burago, Y., Gromov, M., and Perelman, G. (1992), A.D.Alexandrov's spaces with curvature bounded below, Russian Maths Surveys, Uspechi Mat. Nauk., 42(2), 1-58.
- [44] Glandsdoff, P. and Prigogine, I., (1971), Thermodynamic theory of structure, Stability and fluctuations Wiley interscience: London
- [45] Oldham, K.B. and Spanier, J.S. (1974), The fractional calculus, Academic Press: New York.