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On the Explicit Scheme with Variable Time Steps for Solving the Parabolic Optimal Control Problem

A. D. Romanenko*

Kazan Federal University, Kazan, 420008 Russia

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Abstract—The optimal control problem including a linear parabolic equation as the state problem is considered. Pointwise constraints are imposed on the control function. The objective functional contains a given observation function on the entire domain at each moment of time. The optimal control problem is approximated by a finite-dimensional problem with grid approximation of the state equation by using an explicit scheme with variable time steps. The existence and uniqueness of solutions for the continuous and grid optimal control problems are proved. The finite-dimensional optimal control problem is solved by the Udzawa iteration method. Results of numerical experiments are presented.

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INTRODUCTION

Important problems being studied at present include the parabolic optimal control problems. On the whole, as rightly mentioned in [1], parabolic problems with pointwise constraints on the state, unlike elliptic ones, have been little studied and have scarce literature as yet. For example, we mention book [2], in which not only the existence of a solution of the problem was theoretically substantiated but also solution algorithms based on an implicit approximation of the state equation by using gradient methods were described. We also mention [3, 4], where the convergence of iteration methods for the problem with a constraint on the time derivative of the state function was investigated.

This paper is devoted to the numerical solution of the optimal control problem in which the state problem is described by a parabolic equation. The control and observation functions are distributed in the entire domain at each moment of time; constraits are imposed only on the conrol. The approximation of the state equation is based on applying the finite element method to a spatial operator and replacing the time derivative by an explicit difference relation with variable time steps. In this case, there is no necessity to solve a system of linear algebraic equations on each layer, which substantially reduces computational time. A similar problem with constraints imposed not only on the control function but also on the system state and a (distributed or final) was studied in [5], where constant-step approximation was performed. On the other hand, the problem which we consider involves technical difficulties; namely, it is required to store a large number of intermediate results calculated at each moment of time. This difficulty can be partially overcome by appropriately choosing time steps, which makes it possible to perform computations consuming a relatively small amount of memory without the loss of accuracy. A similar situation occurs when implicit approximation, which is known to be absolutely stable, is used.

In this paper, we successfully implement the numerical method of Udzawa [6–8]; the rate of convergence of this method does not depend on the grid parameters. It is worth mentioning that computation by explicit formulas imposes strict constraints on the choice of time steps [9, 10] both in the case of a grid with constant step and in the case of a grid with variable step. Nevertheless, there is a algorithm for reordering steps, which prevents the accumulation of rounding errors. The existence and

*E-mail: romart92@mail.ru