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On One Functional-Differential Quasilinear Hyperbolic Equation

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Abstract—For functional-differential quasilinear hyperbolic equation not investigated earlier, we consider problems with conditions on characteristics and obtain conditions of solvability.

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A lot of existing publications is dedicated to the study of ordinary differential equations, in which arguments of the sought-for function are under some shift. Various authors give various names to the equations under discussion: functional-differential, with retarded or deviating argument (see bibliography in [1]). For partial differential equations there is a small number of publications. Here we can only refer to a few results (e.g., [1–7]). In this paper we consider a new variant of realization of a shift in application to quasilinear hyperbolic equation $u_{xy}(x,y) = f(x,y,u,u_x,u_y)$. Problems for this equation were studied, e.g., in [8] (P. 292), [9] (P. 205). In the present paper, the idea from [1], [4–6], earlier implemented in hyperbolic equation with a shift of arguments in the sought-for function and equation with pseudoparabolic operators of the third and the fourth order, is extended to quasilinear hyperbolic equation.

1. In the domain $D = \{0 < x, y < 1\}$, consider the equation

$$\alpha_1(x,y)u_{xy}(x,y) + \alpha_2(x,y)v_{xy}(x,y) = f(x,y,u,u_x,u_y,v,v_x,v_y) \tag{1}$$

(the author is unaware of the studies of (1) for $\alpha_2 \neq 0$ in the works of other investigators). Here $v(x,y)$ is determined by $u(x,y)$ by the formula

$$v(x,y) \equiv u(y,x). \tag{2}$$

Coefficients in (1) satisfy conditions $\alpha_i \in C^{1,1}(\overline{D})$, $i = 1,2$ (here $C^{k,l}$ is a class of functions with continuous derivatives $\frac{\partial^{i+j}}{\partial x^i \partial y^j}$ for all $0 \le i \le k$, $0 \le j \le l$). The function f is such that

1) it is continuous by all of its arguments, if $(x,y) \in D$ and $|u| \le A_1$, $|u_x| \le A_2$, $|u_y| \le A_3$, $|v_x| \le A_4$, $|v_y| \le A_5$, where $A_j > 0$, $j = 1,\dots,5$, are known numbers;

2) it satisfies the Lipschitz condition

$$|f(x,y,u_1,u_{1x},u_{1y},v_1,v_{1x},v_{1y}) - f(x,y,u_2,u_{2x},u_{2y},v_2,v_{2x},v_{2y})|$$
$$\le k\big(|u_1-u_2| + |u_{1x}-u_{2x}| + |u_{1y}-u_{2y}| + |v_1-v_2| + |v_{1x}-v_{2x}| + |v_{1y}-v_{2y}|\big),$$

$k = \text{const} > 0$.

Goursat problem (Γ**).** *Find in D a function $u \in C^{1,1}(D) \cap C(\overline{D})$, being a solution to Eq.* (1)*, and satisfying the conditions*

$$u(0,y) = \varphi(y), \quad \varphi \in C^1[0,1], \tag{3}$$

$$u(x,0) = \psi(x), \ \ \psi \in C^1[0,1], \tag{4}$$

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