ISSN 1066-369X, Russian Mathematics, 2017, Vol. 61, No. 12, pp. 73–79. © Allerton Press, Inc., 2017.
Original Russian Text © D.T. Tapkin, 2017, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2017, No. 12, pp. 84–91.

Isomorphisms of Formal Matrix Incidence Rings

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Received April 11, 2017

Abstract—In a paper published in 2008 P. A. Krylov showed that formal matrix rings $K_s(R)$ and $K_t(R)$ are isomorphic if and only if the elements s and t differ up to an automorphism by an invertible element. Similar dependence takes place in many cases. In this paper we consider formal matrix rings (and algebras) which have the same structure as incidence rings. We show that the isomorphism problem for formal matrix incidence rings can be reduced to the isomorphism problem for generalized incidence algebras. For these algebras, the direct assertion of Krylov's theorem holds, but the converse is not true. In particular, we obtain a complete classification of isomorphisms of generalized incidence algebras of order 4 over a field. We also consider the isomorphism problem for special classes of formal matrix rings, namely, formal matrix rings with zero trace ideals.

DOI: 10.3103/S1066369X1712009X

Keywords: *formal matrix ring, incidence algebra, generalized incidence algebra, isomorphism problem, upper triangular matrix ring, zero trace ideals.*

INTRODUCTION

All rings in question are assumed to be associative rings with unity, modules and bimodules are assumed to be unitary. The Jacobson radical, the center, and the group of invertible elements of a ring R are denoted, respectively, by $J(R)$, $C(R)$, and $U(R)$.

Incidence algebras were introduced in mid-60s as a natural tool for the study of combinatorial problems. Soon after it became clear that these algebras are of interest on their own account. In particular, they include the product of n copies of a ring R and the ring of upper triangular matrices over R. There is a close association of incidence algebras with subalgebras of matrix rings over fields. Incidence algebras can be viewed as a particular case of formal matrix rings. Rings of formal matrices of order 2 are often called Morita context rings.

In the study of formal matrix rings, there naturally arises a construction which generalizes incidence algebras. We call it here generalized incidence algebra. We consider formal matrix rings all of whose bimodules are either zero or a fixed ring regarded as a bimodule over itself. One can easily see that in the case of a commutative ring this construction is also an algebra. The isomorphism theory developed in this paper for such algebras made it possible to consider the problem on conversion of the theorem on isomorphism from [1] for matrices of large orders and, in particular, to get the complete classification of isomorphism classes of generalized incidence algebras of order 4 over fields. This theory finds application in the isomorphism problem for formal matrix rings with values in a commutative local ring as well as for formal matrix incidence rings with values in a commutative local ring.

The techniques developed in the proof enables one to get applications for upper triangular formal matrix rings and for formal matrix rings with zero trace ideals.

In [2] and [3] the isomorphism problem was studied in detail for upper triangular formal matrix rings of order 2 and n, respectively. In [4], a similar theorem was obtained for Morita context rings with zero trace ideals. Though these papers give necessary and sufficient conditions, these conditions are presented in implicit form.

In this paper we obtain necessary and sufficient conditions in explicit form for upper triangular formal matrix rings of order n as well as for formal matrix rings of order n with zero trace ideals.

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