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Two-Level Iterative Method for Non-Stationary Mixed Variational Inequalities

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Abstract—We consider a mixed variational inequality problem involving a set-valued non-monotone mapping and a general convex function, where only approximation sequences are known instead of exact values of the cost mapping and function, and feasible set. We suggest to apply a two-level approach with inexact solutions of each particular problem with a descent method and partial penalization and evaluation of accuracy with the help of a gap function. Its convergence is attained without concordance of penalty, accuracy, and approximation parameters under coercivity type conditions.

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PRELIMINARIES

Let D be a nonempty set in the real n-dimensional space $\mathbb{R}^n$, $h : D \to \mathbb{R}$ a convex function, and let $G : D \to \Pi(\mathbb{R}^n)$ be a point-to-set mapping. Here $\Pi(A)$ denotes the family of all nonempty subsets of a set A.

Then one can define the *mixed variational inequality problem* (MVI, for short), which is to find an element $x^* \in D$ such that

$$\exists g^* \in G(x^*),\ \langle g^*, y - x^*\rangle + h(y) - h(x^*) \geq 0 \quad \forall y \in D. \tag{1}$$

Suppose also that D is a set of the form

$$D = V \bigcap W, \tag{2}$$

V and W are convex and closed sets in the space $\mathbb{R}^n$. This partition of the feasible set is optional and usually means that V represents "simple" constraints whereas W corresponds to complex or "functional" ones and a suitable penalty function should be used for this set.

Problem (1) was first proposed in [1], [2] (with the single-valued mapping G) and further investigated by many authors (see, e.g., [3–7]). MVIs give a suitable format for various problems arising in Economics, Mathematical Physics, and Operations Research. Besides, the usual variational inequalities and convex optimization problems can be viewed as particular cases of MVI (1).

We observe that most existing solution methods for these problems require exact values of the cost mapping G, function h, and feasible set D. However, this is often impossible due to the calculation errors and lack of the necessary information. The same situation arises if we find it useful to replace the initial problem by a sequence of auxiliary ones with better properties, as in regularization and penalty methods. Within this approach, we can also replace general nonlinear functions with their simple (for example,

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