

# Symmetry accounting in the integral-equation analysis of lasing eigenvalue problems for two-dimensional optical microcavities

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## Abstract

© 2017 Optical Society of America. We consider the modes of two-dimensional (2D) microcavity lasers as active open dielectric resonators using the linear electromagnetic formalism of the lasing eigenvalue problem (LEP) with exact boundary and radiation conditions. We reduce LEP to a nonlinear eigenvalue problem for the Muller boundary integral equation and build a sophisticated numerical method accounting for the possible symmetry properties of sought solutions. Namely, we take into account the presence of one, two, or four lines of symmetry. This helps split solutions into independent classes, which contributes to the stability of calculations and reduces the size of the approximate matrix eigenvalue problems.

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