

Degrees of Categoricity of Rigid Structures

Nikolay A. Bazhenov^{1,2}  and Mars M. Yamaleev³ 

¹ Sobolev Institute of Mathematics, Novosibirsk, Russia

² Novosibirsk State University, Novosibirsk, Russia

`bazhenov@math.nsc.ru`

³ Kazan (Volga Region) Federal University, Kazan, Russia

`Mars.Yamaleev@kpfu.ru`

Abstract. We prove that there exists a properly 2-c.e. Turing degree \mathbf{d} which cannot be a degree of categoricity of a rigid structure.

Keywords: Categoricity spectrum · Strong degree of categoricity · Rigid structure · 2-c.e. Turing degrees

1 Introduction

The study of effective categoricity for computable structures goes back to the works of Fröhlich and Shepherdson [1], and Mal'tsev [2,3]. In recent years, the focus of the research in the area is on computable categoricity relative to Turing degrees.

Definition 1. Let \mathbf{d} be a Turing degree. A computable structure \mathcal{A} is \mathbf{d} -computably categorical if for every computable copy \mathcal{B} of \mathcal{A} , there is a \mathbf{d} -computable isomorphism from \mathcal{A} onto \mathcal{B} . The categoricity spectrum of \mathcal{A} is the set

$$\text{CatSpec}(\mathcal{A}) = \{\mathbf{d} : \mathcal{A} \text{ is } \mathbf{d}\text{-computably categorical}\}.$$

A Turing degree \mathbf{d} is the degree of categoricity of \mathcal{A} if \mathbf{d} is the least degree in the spectrum $\text{CatSpec}(\mathcal{A})$.

Categoricity spectra and degrees of categoricity were introduced in [4]. Suppose that n is a natural number and α is an infinite computable ordinal. Fokina, Kalimullin, and Miller [4] proved that each Turing degree \mathbf{d} that is 2-c.e. in and above $\mathbf{0}^{(n)}$ is the degree of categoricity for a computable structure. Csima, Franklin, and Shore [5] extended this result to hyperarithmetical degrees. They

N.A.Bazhenov—Supported by RFBR project No. 16-31-60058 mol.a.dk.

M.M.Yamaleev—Supported by RFBR projects No. 15-01-08252, 16-31-50048, and by the subsidy allocated to Kazan Federal University for the state assignment in the sphere of scientific activity (No. 1.1515.2017/PCh). The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University and by the research grant of Kazan Federal University.