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# Viscous cosmology for early-And late-Time universe

Brevik I., Grøn Ø., De Haro J., Odintsov S., Saridakis E.
*Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia*

## Abstract

© World Scientific Publishing Company. From a hydrodynamicist's point of view the inclusion of viscosity concepts in the macroscopic theory of the cosmic fluid would appear most natural, as an ideal fluid is after all an abstraction (exluding special cases such as superconductivity). Making use of modern observational results for the Hubble parameter plus standard Friedmann formalism, we may extrapolate the description of the universe back in time up to the inflationary era, or we may go to the opposite extreme and analyze the probable ultimate fate of the universe. In this review, we discuss a variety of topics in cosmology when it is enlarged in order to contain a bulk viscosity. Various forms of this viscosity, when expressed in terms of the fluid density or the Hubble parameter, are discussed. Furthermore, we consider homogeneous as well as inhomogeneous equations of state. We investigate viscous cosmology in the early universe, examining the viscosity effects on the various inflationary observables. Additionally, we study viscous cosmology in the late universe, containing current acceleration and the possible future singularities, and we investigate how one may even unify inflationary and late-Time acceleration. Finally, we analyze the viscosity-induced crossing through the quintessence-phantom divide, we examine the realization of viscosity-driven cosmological bounces, and we briefly discuss how the Cardy-Verlinde formula is affected by viscosity.

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## Keywords

Dark energy, Inflation, Modified gravity, Viscous cosmology

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